Superior Solutions for Electromagnetic Field Designs:
The Boundary Element Method

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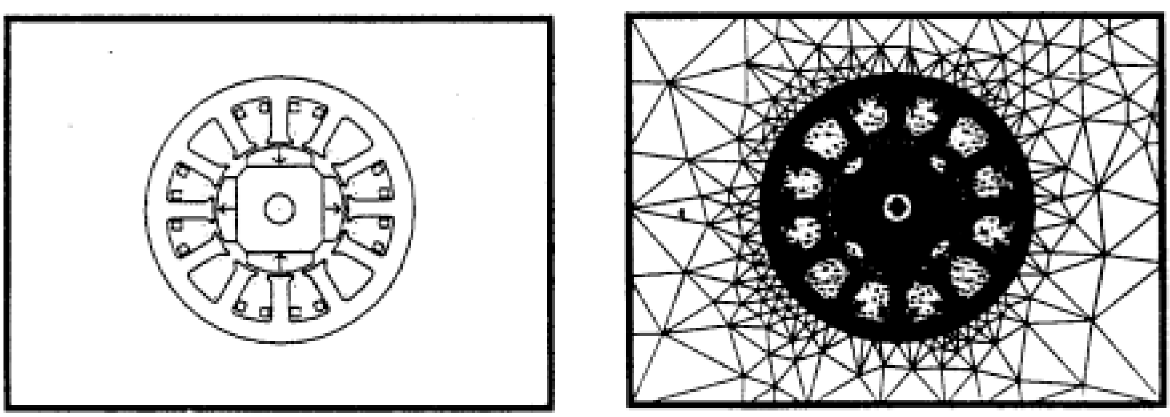
As electromagnetic field simulation enters the mainstream of computer-aided engineering (CAE), the boundary element method (BEM) is emerging as the superior solution technique, thereby giving manufacturers a significant competitive advantage.

What is BEM?

The Boundary Element Method (BEM) is a numerical technique for solving Boundary Integral Equations. In this technique, electromagnetic phenomena are mathematically described by Maxwell's equations in **integral** form. Enforcing the boundary conditions along the material interfaces allows one to obtain a set of boundary integral equations with the unknowns as the equivalent sources or field variables along the interfaces. One may then discretize the boundaries to boundary elements, represent the unknowns on elements, and obtain a system of linear equations. In this way BEM solves a given problem. All field variables at any point in space may be obtained by performing integrations associated with the equivalent sources or fields on the boundaries.

What is the basic difference between BEM and FEM?

Unlike the BEM, the Finite Element Method (FEM) is a numerical technique for solving Maxwell's equations in **differential** form. For a given design, the FEM requires the entire design, including the surrounding region, to be modelled with finite elements. A system of linear equations is generated to calculate the potential (scalar or vector) at the nodes of each element. Therefore, the basic difference between these two techniques is the fact that BEM only solves the unknowns on the boundaries, whereas FEM solves for the whole space (Figure 1).



BEM **FEM**

Figure 1. 2D Motor Example - BEM model versus FEM model

BEM and FEM have become the two dominant numerical techniques in computer-aided engineering (CAE). Both techniques have merits and restrictions. However, in comparing the many aspects of both techniques, Integrated Engineering Software concludes that BEM is the superior solution technique for electromagnetic field designs.

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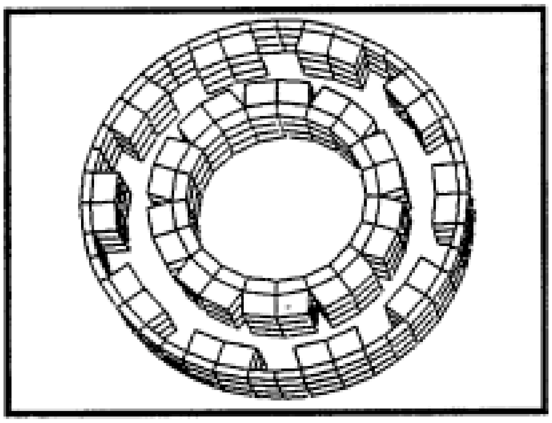
What are the major advantages of BEM?

Ease of Use.

Unlike FEM, which must use a 3D finite element mesh in the whole space, BEM uses only 2D elements on the surfaces which are the material interfaces or assigned boundary conditions (Figure 2). Therefore, users can set up a problem quickly and easily. Since only elements on interfaces are involved in the solution procedure, problem modifications are also easy. For example, in motor design optimization, solutions are required for different rotor positions. Using BEM software, only one boundary element distribution is necessary to solve all the rotor positions, and no element reassignments are required. With FEM software, finite elements in the whole space must be re-generated for every new rotor position.



Complete 3D Finite Element Meshes are impossible to visually represent and comprehend on a 2D drawing.

BEM - surface mesh **FEM - volume mesh**

Figure 2. 3D Magnetic Clutch Example - BEM model versus FEM model

More accurate results.

BEM allows all field variables at any point in space to be obtained very accurately. Also, the results are more precise because the integration operation is smoother, making BEM inherently more accurate than FEM's differentiation operation. Moreover, the unknown variables used in INTEGRATED's software are the equivalent currents or equivalent charges. These variables have real physical meanings. By using these physical variables, global quantities such as forces, torque, stored energy, inductance, and capacitance among others can be accurately obtained through some very simple methods.

Superior analysis of open boundary problems.

The analysis of unbounded structures (e.g. electromagnetic fields exterior to an electric motor) can be solved by BEM without any additional effort because the exterior field is calculated the same way as the interior field. The field at any point in space can be calculated (even at infinity). Therefore, for any closed or open boundary problem INTEGRATED's software users need only to deal with real geometry boundaries. In contrast, open boundary problems are problematic for FEM since artificial boundaries, which are far away from the real structure, must be used. How to determine these artificial boundaries becomes a major difficulty for FEM-based software users. Since most electromagnetic field problems are associated with open boundary structures, BEM naturally becomes the best method for general field problems.

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Is it true that matrix solution requires less time and memory with FEM because FEM generates a sparse matrix, whereas BEM has a full matrix?

It is true that FEM requires less time and memory than BEM to solve matrices of the same size. However, it is also true that BEM always generates a much smaller matrix than FEM for a given problem. As mentioned earlier, BEM places the unknowns only on the boundaries while FEM has to use unknowns in the entire space. For a closed boundary problem, in general the unknowns are N^2 with BEM and N^3 with FEM, where N is a constant. For an open boundary problem, FEM will generate a much larger matrix. Also, INTEGRATED uses an iterative method to solve the matrix in our software. Therefore, the time of solving an MxM matrix in INTEGRATED's software is proportional to M^2. Although it is difficult to compare the solution speeds between BEM and FEM, we conclude from our experience with these methods that our programs are generally faster than programs using FEM.

Can BEM solve non-linear problems?

BEM can readily solve non-linear problems. Many people, including some who are quite knowledgeable about BEM initially, believed that BEM could not solve non-linear problems. However, the truth is that not many people know how to use BEM for solutions to non-linear problems. Later on we will discuss in more detail how BEM is used to solve non-linear problems. For now it is sufficient to point out that BEM routinely solves non-linear problems.

Is post processing faster with BEM or FEM?

Calculating the field distributions with FEM is faster than with BEM. The reason is that FEM calculates the field value using differential operations on the potential variables, while BEM has to perform an integration along the boundaries. However, since the differentiation introduces discontinuities, it is difficult for FEM to obtain accurate field quantities by using simple direct differentiation. In contrast, with BEM all field quantities are derived by integration which is inherently stable.

Therefore, although BEM takes longer to calculate the fields, it gives more accurate results. Tremendous effort has been made at INTEGRATED to ensure that most integrations in our 2D/RS software are performed analytically. This greatly increases the speed and accuracy of INTEGRATED's software.

Can BEM do error analysis?

Yes. From Green's theorem one can show that if and only if the solution satisfies the boundary conditions on all the boundaries, the result at any point in the solution space obtained from the variables on the boundaries is correct. Therefore, after solving a problem with certain element distribution, users can perform an error analysis by checking the boundary conditions along the boundaries. One can improve the solution by simply adding more elements on the boundary where a large error has been found. This is based on the fact that the largest errors occur on the boundary, and that for the fields in a region the largest contributors are the elements close to the region. We have an error detection command in all of our 2D/RS software.

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I can see that BEM is the superior solution technique; however, there seems to be more commercial FEM programs available and FEM has been more familiar to electrical engineers... why?

Until recently, largely due to historical reasons and the relative simplicity of implementation, FEM received the most research attention. In addition, CAE programming was simply much easier to do with FEM (provided a mesh can be generated). **The single largest difficulty in developing BEM packages for electrical engineering lay not in the basic theory, but in translating its complexity into software easily used by the design engineers.** BEM's complexity, and the numerical difficulties associated with the integral equation theory, deterred many programmers from the daunting task. Within the last several years, however, as these theoretical challenges have been worked through, significant hurdles have been overcome. **While the actual programming remains very difficult, the resulting BEM software packages have become very usable and highly desirable tools.** Problems previously believed to be practically insurmountable, such as non-linear problems, are now easily solved.

How and why can BEM solve non-linear problems?

Many papers have been published in recent years. Here, we only list a sample for your reference.

1. Meng H. Lean and Dan S. Bloomberg, "Non-linear boundary element method for two-dimensional magnetostatics," *J. Appl. Phys.* 55(6), pp.2195-2197, 15 March 1984.

2. R. Shao and K. D. Zhou, "The iterative boundary element method for non-linear electromagnetic field calculations," *IEEE Trans. on Magnetics*, Vol. 24, pp.150-153, 1988.

3. Krstajic, Z. Andelic and S. Salon, "Nonlinear 3D magnetostatic field calculation by the integral equation method with surface and volume magnetic charges," *IEEE Trans. on Magnetics*, Vol. 28, pp.1088-1091, 1992.

4. A.J. Nowak, and A.C. Neves, "The Multiple Reciprocity Boundary Element Method", Computational Mechanics Publications, p 240, 1994.

Without delving into a detailed discussion of Maxwell's equations, non-linearity for magnetostatics may be explained as follows:

Basic field theory provides that any magnetized body, such as magnets and iron, will produce a magnetic field which can be exactly modelled by a set of equivalent surface and volume currents on and within the structure. This is true whether the materials are linear or non-linear. In cases where it is assumed that the materials are linear, it can easily be shown that the equivalent volume currents will be zero and that only the equivalent surface currents will be present.

For linear problems, it is precisely these equivalent surface currents which are calculated in our magnetostatic problems. A matrix of linear equations are generated from the boundary elements. The matrix is subsequently solved to determine the equivalent approximation to the surface currents. Any field parameter can then be found via straightforward integration.

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For non-linear problems, BEM is still applicable with a little modification. In this case, however, the method must be expanded to deal with the equivalent volume currents. The field produced by the volume currents is small compared to that of the surface currents, and for many practical problems it can simply be neglected. If, however, volume currents are significant, the regions changing rapidly from an unsaturated to a saturated state need to be discretized. In 2D/RS and 3D programs this is accomplished by generating subareas and subvolumes in these respective regions. The equivalent volume currents are found by an iterative scheme and are put in the right hand side of the system of equations, rather than in the system matrix. This is a very important distinction. Other methods that deal with non-linear problems using integral equations require that the volume unknowns appear in the system matrix. This results in a volume integral formulation, as opposed to the boundary integral formulation used in all INTEGRATED software. Volume integral formulations usually result in significantly larger matrices and longer solution times. Other techniques, such a multiple layers of boundary elements to account for non-linearities have also been developed.

As the system matrix size is unaffected by the equivalent volume currents, the method used in INTEGRATED's software is still a boundary element method with the additional volume unknowns appearing in the right hand side.

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BEM versus FEM

Summary

	IES BEM	*Conventional BEM*	*FEM*
Modeling (3D)			
1. Physical Geometry	exact geometry	linear or quadratic fit	artificial discontinuities (at element edges)
2. Mesh	Surface - easy production and inspection	Surface - easy production and inspection	Volume - complex to produce, inspection difficult
3. Size - Mesh Points	N^2	N^2	N^3
4. Open Boundary Conditions	automatically satisfied - for source balance or zero potential at infinity	automatically satisfied - for zero potential at infinity	artificial boundaries
5. Non-Linear Material	subareas only where saturation occurs - general solution	layered surface mesh	unique values for each element
Analysis			
1. Solution Method	Integral equation formulation, interative solver, Galerkin	Integral equation formulation LUD decomposition MOM - limited applications	Partial differential equations - introduces discontinuities
2. Initial Solution	field source	field source	potential (scalar or vector)
3. Parametric Solutions	no remeshing	no remeshing	must remesh
4. Post Processing	integrating - inherent stability	integrating - inherent stability	potential derivatives - extreme care must be taken for derivatives
5. Solution Time	kN^4 diagonally strong matrix	N^6 full matrix	$N^{3\,5 \text{ to } 4\,5}$ sparse matrix
6. Error Detection	natural	natural	complex
7. Storage • **temporary** • **final solution**	 N^3 N^2	 N^3 N^2	 N^3 N^3