Advantages of NURBS in CAE Modeling

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In Mar. 2002 we released version 6.0 of our CAE software. The following article is written to explain why we felt it was important to switch our geometric handling to NURBS.

Advantages of NURBS in CAE Modeling

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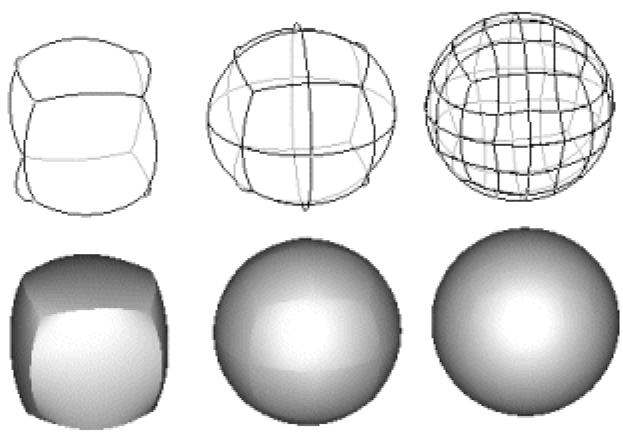
ABSTRACT

Integrated Engineering Software has just completed years of development with the introduction of NURBS replacing the Coons Patch as the basis for the geometric aspect of their modeling. This makes it easier to accurately model general 3D curved surfaces and enables direct links to CAD packages.

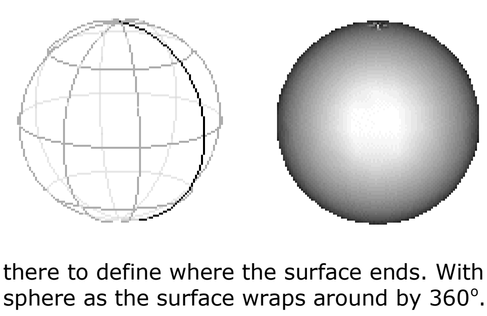
To explain why we felt NURBS would represent a significant improvement to our CAE simulation software, we will start by reviewing our old geometric modeling using Coons Patches.



What are *Coons Patches*?

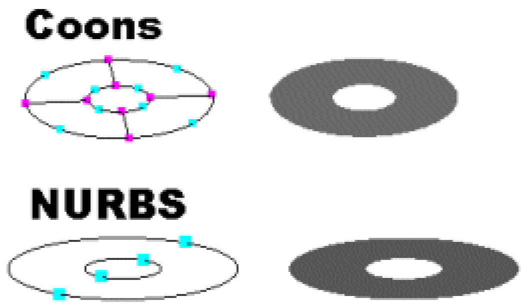
To left is shown three Coons Patch constructions of a sphere - in wire frame and shaded views. They consist of several surfaces each of which approximates a small part of the sphere. The Coons Patch is a method for finding a surface from the curves which form its boundary. There are, of course, an infinite number of possibilities, but the Coons Patch can be thought of as the surface obtained from a soap film over a wire frame of the given shape. On the far left the sphere made from 6 surfaces amounts to a slightly rounded cube. Subdividing each of these in 4 with appropriate arcs pushes the surfaces outwards and provide a much better approximation to a sphere. This 24 surface sphere has been our standard sphere primitive prior to version 6. Customers requiring better definition could further subdivide the sphere surface as shown by the 96 surface patch.

Simple Coons Patches can give very good approximations to surfaces that only curve in two dimensions (e.g. a cylinder), but general 3D objects require some thought and work to construct a good surface. Further down the page we will illustrate the effect on the accuracy of the physical models, but first an explanation of our replacement to the Coons Patch - NURBS.



What are *NURBS*?

Non-**U**niform **R**ational, **B**-splines.
NURBS permit definition of surfaces from ratios of polynomials. (Rational functions permit much better control over the derivatives of curves, hence the surface curvature, than polynomials alone.) Our new sphere primitive is shown to the left in wire frame and shaded views. There is only one segment in this case, the black 180° arc on the right, gray arcs are only there to visualize the sphere. With NURBS the surface exists by definition, segments are there to define where the surface ends. With the sphere, the one arc segment defines the two "edges" of the sphere as the surface wraps around by 360°.



"Holey" Surfaces

Another class of geometry where the NURBS make modeling much faster and easier is surfaces/volumes with holes. The Coons Patch is a grid defined from the outside edges - there is no allowance for holes in the middle. A NURBS surface can be trimmed with segments that carve holes out of the inside. So something as simple as a circle with a hole in it requires a patchwork of surfaces around the hole using a Coons Patch, but only a plane surface trimmed by two arcs for NURBS.

NURBS & CAD

For years major CAD software has been using NURBS for good surface definition. However, the exchange formats IGES and DXF are more limited - hence, the original model can become somewhat distorted in transfering from one program to another. Add that to some inconsistency in how these formats are defined and consider a task like importing the NURBS sphere into a Coons Patch based program... prior to version 6.0 there were many places where things could go wrong for our customers in getting their CAD drawings into our software. With NURBS it is now possible to represent the geometry the same as the CAD packages represent it internally - so rather than using faulty exchange formats we have partnered with the major CAD vendors and now have a direct link that will pull a model from memory in the CAD program into one of our programs.

An Illustration of the Effect on the Accuracy of Results

One of our standard benchmarking tests on the electrostatic software is a small point charge near a grounded sphere. This is very fast to construct, and we invite any readers with other 3D electrostatic modeling packages to compare their results with what we report below:

The model consists of:

- a 1 m radius sphere set at zero volts, located at x=y=z=0
- a 0.001 m radius sphere with a total charge of 1 microCoulomb located at x=z=0,y=1.5m

Using the method of images it can be shown that outside the larger sphere this is equivalent to the superposition of a 1 microCoulomb located at x=z=0,y=1.5 m and a -2/3 microCoulomb charge located at x=z=0,y=2/3 m. Here is an easy to construct model whose extreme geometric ratios will present some challenge, but which also has an analytic answer to compare with any results.

v5.2 (Coons Patch) Results
This model was set up in COULOMB5.2 using the standard 24 surface primitive sphere, with 1500 elements on a Pentium III and took about a minute to solve. The table below presents some randomly selected results:

v5.2 - Some Randomly Selected Locations - 24 Surface Patches					
x (m)	y (m)	z (m)	Analytic Answer	COULOMB5.2 Answer	% Difference
2	0	0	753.11	763.55	1.4%
0	0	1.5	586.74	597.70	1.9%
20	0	0	148.74	150.09	0.91%
20	100	50	27.040	27.281	0.89%
200	3000	70	0.99745	1.0086	1.1%
** this table is revisited and "corrected" at the end					

Given the many orders of magnitude change between parts of the model and between the positions chosen to calculate V, one might think a 1% discrepancy is quite good. However, we can do much better if we refine the surface. When the spheres are broken up into 96 surfaces each the analysis still only takes a minute with 1500 elements, however the results become:

v5.2 - Some Randomly Selected Locations - 96 Surface Patches					
x (m)	y (m)	z (m)	Analytic Answer	COULOMB5.2 Answer	% Difference
2	0	0	753.11	753.93	-0.11%
0	0	1.5	586.74	586.37	0.063%
20	0	0	148.74	148.84	-0.063%
20	100	50	27.040	27.056	-0.059%
200	3000	70	0.99745	1.0002	-0.28%

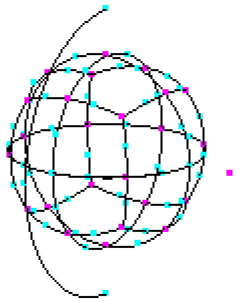
Better surface definition has reduced the discrepancy between COULOMB and the analytic result by a factor of 10. This is powerful motivation for using NURBS in order to have much better surface definition still, and to have it with less work.

With COULOMB6.0 the spheres consist of a single surface each, and the analysis time is actually shortened a bit (fewer surface seams means fewer unknowns, even for the same total number of elements). The results become:

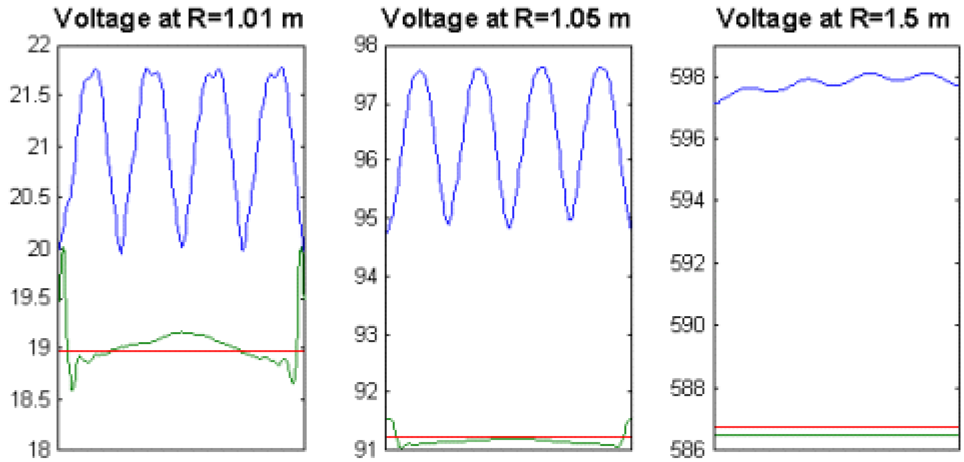
v6.0 - Some Randomly Selected Locations - 1 Surface					
x (m)	y (m)	z (m)	Analytic Answer	COULOMB6.0 Answer	% Difference
2	0	0	753.11	752.82	0.039%
0	0	1.5	586.74	586.50	0.041%
20	0	0	148.74	148.69	0.034%
20	100	50	27.040	27.030	0.036%
200	3000	70	0.99745	0.99709	0.036%

The results are improved again, and of the three models this was the easiest to construct and the fastest to solve.

A Closer Examination of the Benefits



Calculating V along equipotential arcs that are symmetric around the symmetry axis of the problem, we are able to get better statistical characterization of expected results for the three models introduced above. The root of the problems from poor surface definition are best seen close to the spheres, so radii of 1.5, 1.05, & 1.01 m were chosen. In the graphs below the blue curves are the 24 surface COULOMB5.2 model, the green curves are the COULOMB6.0 result, and the red lines are analytic expected values.



The R=1.01 m plot shows the basic limitation prior to version 6.0 - a curve very close to the large sphere shows oscillations as the sphere surface stetches inwards and outwards slightly at different locations. Furthermore, since the average radius is not quite correct the overall average it is displaced a more than for version 6.0. In version 6.0, the limitation for a near curve is the density of the elements and the chosen level of integration accuracy. Particularly problematic in this quick test is the mapping of elements around the singularities at the poles of the sphere. However, the worst cases with version 6.0 are approximately the same as the best cases with version 5.2 - the geometrical limitation is overcome and if a more accurate result is desired we have options to find it.

The other radii show the same basic effects, however further out from the spheres the effects become relatively smaller. The numeric results are summarized below.

Statistics for 3 Small Radii			
R (m)	Analytic Answer (V)	v5.2 - 24 Surfaces	v6.0 - 1 Surface
1.01	18.989	21.041 error: 10.8% std: 0.62	19.044 error: 0.29% std: 0.22
1.05	91.217	96.435 error: 5.7% std: 0.96	91.168 error: 0.054% std: 0.097
1.5	586.74	597.78 error: 1.88% std: 0.23	586.50 error: 0.042% std: 0.0029

COULOMB5.2 Revisited

At this point the reader may still be confused about the COULOMB5.2 - 24 surface sphere. The farther out we find the results, the less the imperfections in the surface definition matter - hence the smaller the standard deviation found in V. However, the average value of V seems to get no better than about a 1% discrepancy compared to the analytic result, even though the 96 surface sphere is much better. Why isn't there a benefit from greater distance?

In fact, the "1 m radius sphere" is composed of surfaces bounded by arcs of 1 m radius. Examine the shaded view of this surface at the start of this article, and it becomes clear that this is the maximum radius of any part of the sphere. The flattening of the patches leads to a smaller average radius - R=0.99574 m. This is about 0.4% lower than what was used in the calculations above. Below is a corrected version of the first table:

v5.2 - Some Randomly Selected Locations - 24 Surface Patches Corrected for R=0.99574					
x (m)	y (m)	z (m)	Analytic Answer	COULOMB5.2 Answer	% Difference
2	0	0	762.82	763.55	0.095%
0	0	1.5	597.21	597.70	0.082%
20	0	0	150.01	150.09	0.050%
20	100	50	27.268	27.281	0.046%
200	3000	70	1.0059	1.0086	0.26%

Clearly the limitation in COULOMB5.2 was the difficulty in accurately knowing the average radius of the sphere being modeled. More accurate results required more work on the surfaces - such as using a 96 surface sphere. COULOMB6.0 gives you this benefit for no extra work.

CONCLUSION

The switch to NURBS modeling in version 6.0 means that general 3D curved geometries are not only easier to draw, but will also tend to solve more accurately and a bit faster than the version 5.2 equivalent models.

A caution to users upgrading to version 6.0 - it will read your old models into exactly the same structure as you drew it. We do not make any guesses about what may have been intended by any given patchwork of surfaces. If you want the benefits of better surface definition for an existing model you should redraw the parts of interest.

COULOMB6.0 users who wish to verify the numbers cited in this article can **download the model** and make their own observations. You can also construct the model yourself in a couple of minutes. Users of other 3D electrostatic simulation software are invited to reproduce this model and compare the speed of modeling and solving as well as the solution accuracy with the numbers cited above for COULOMB6.0.