Simulation of Polyphase Induction Machines: Frequency Domain Boundary Element Method

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This paper describes a frequency domain Boundary Element Method (BEM) for simulating and analyzing polyphase induction machines with steady-state sinusoidal time-varying current source excitation.
A dual simple-layer boundary integral equation and its numerical solution are presented. Advantages of the BEM technique, such as ease of use, accurate solutions, and significant reduction in computation time, are addressed. A sample analysis is illustrated and some numerical results given for a three-phase induction motor with current excitation to demonstrate the use of BEM software in the design and analysis of the polyphase induction machine. The possibility of analyzing and induction motor with voltage excitation is also outlined.

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INTEGRATED ENGINEERING SOFTWARE

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This paper describes a frequency domain Boundary Element Method (BEM) for simulating and analyzing polyphase induction machines with steady-state sinusoidal time-varying current source excitation. A dual simple-layer boundary integral equation and its numerical solution are presented. Advantages of the BEM technique, such as ease of use, accurate solutions, and significant reduction in computation time, are addressed. A sample analysis is illustrated and some numerical results given for a three-phase induction motor with current excitation to demonstrate the use of BEM software in the design and analysis of the polyphase induction machine. The possibility of analyzing and induction motor with voltage excitation is also outlined.

Introduction

With the advent of high speed microcomputers, numerical methods for simulating and analyzing electromagnetic fields have become the most effective and powerful tools for helping engineers visualize and manipulate EM fields, thereby designing products without the extensive testing and redesign needed to accommodate unforeseen EM effects. The numerical treatment of most linear or nonlinear EM field problems can be effectively achieved through the Boundary Element Method (BEM). As compared to domain-type methods such as the Finite Element Method (FEM), the BEM has two salient advantages: (1) the dimensions of the problem are effectively reduced by one; and (2) the analysis is equally applicable to bounded or unbounded regions. As software based on the BEM has become increasingly popular in the industries, it has proven to be an efficient CAE tool.

This paper describes a 2D frequency domain Boundary Element Method for simulating and analyzing polyphase induction machines with steady-state sinusoidal time-varying current source excitation. A dual simple-layer boundary integral equation and its numerical solution are presented. Advantages of the BEM technique, such as ease of use, accurate solutions, and a significant reduction in computation time, are addressed. A sample analysis is illustrated and some numerical results are given for a three-phase induction motor with current excitation to demonstrate the use of BEM software in the design and analysis of the polyphase induction machine. The possibility of analyzing an induction motor with voltage excitation is also outlined.

Theory

From Maxwell's equation, the magnetic vector potential may be written as:

$$\underline{\nabla} x (\frac{1}{\mu} \underline{\nabla} x \underline{A}) = \underline{J} \qquad (1)$$

where $\underline{J}$ is the current density. In the non-conducting region, the current density is zero; in the stator winding, it is approximately constant and known; and in the conducting bars, it may be expressed as:

$$\underline{J} = \sigma(-j\omega\underline{A} - \underline{\nabla}\Phi + \underline{V} x \underline{\nabla} x \underline{A}) \qquad (2)$$

where Φ the electric potential and $\underline{V}$ is the velocity of the conductor.

By neglecting the edge effects, a two-dimensional analysis can be used for the polyphase induction motor problem. In the 2D model, we assume that only the longitudinal component of vector potential and current density exist, and that the motor is working in the balanced and steady-state condition. Thus, the current density in the conducting bars can be written as

$$J = \sigma(-j\omega \mathrm{A} + \omega_m \frac{\partial \mathrm{A}}{\partial \Phi} \tag{3}$$

where ω and ω_m are the electric and the rotor mechanical angular frequency, respectively. For a polyphase induction motor, we have

$$\omega_m \frac{\partial \mathrm{A}}{\partial \Phi} = (1-S)\frac{2\omega}{P}\frac{\partial \mathrm{A}}{\partial \Phi} \approx j\omega(1-S)A \tag{4}$$

where *S* is the rotor slip and *P* is the number of poles. Thus, one has

$$(\nabla^2 + \kappa^2)A = -\mu J_s \tag{5}$$

where J_s is the source current density and = 0 in the non-conducting region and $\kappa^2 = -j\omega s\mu\sigma$ in the conducting region. Equation (5) may also be obtained by using the argument that from the viewpoint of a rotor observer, he or she sees the fields in the machine changing at the slip frequency.

From the equivalence principle [1,2], one can show that the field at any point may be obtained by using an equivalent surface current distribution on the material interface. We can use a simple-layer current distribution on the interface between two non-conducting regions, and a dual simple-layer current on the conductor surface (one just outside the conductor and the other just inside the conductor). In the non-conducting regions, the field is created from the winding current and all the simple layer equivalent currents in the non-conducting regions. The field in the conductor can be calculated using the surface current just inside the conductor. From equation (5), the magnetic vector potential can be expressed as

$$A(\underline{r}) = \mu \int_s G(\underline{r}, \underline{r}')J(\underline{r}')d\underline{r}' \tag{6}$$

where $\underline{r}$ and $\underline{r}'$ are the field and source points, respectively. In the non-conducting region

$$G(\underline{r}, \underline{r}') = -\frac{1}{2\pi} 1n|\underline{r} - \underline{r}|, \tag{7}$$

while in the conducting region,

$$G(\underline{r}, \underline{r}') = \frac{1}{4j} H_0^{(2)}(\kappa|\underline{r} - \underline{r}'|) \tag{8}$$

where $H_0^{(2)}$ is the zero order Hankel function of the second kind.

By enforcing the A and tangential $\underline{H}$ continuity conditions on the conducting surface, and $\underline{H}$ continuity condition on the interfaces between non-conducting regions, one obtains the following boundary integral equations:

$$\underline{\hat{n}} x[\underline{H}_1(\underline{r}, J) - \underline{H}_2(\underline{r}, J)] = 0 \tag{9}$$

and

$$\underline{A_1(\underline{r}, J)} - \underline{A_2(\underline{r}, J)} = 0 \tag{10}$$

By solving the equations (9) and (10) using the BEM, the equivalent current distribution can be obtained. Other quantities of interest can easily be calculated from the current.

The above analysis is based on the assumption that the field in the motor is not saturated. If we consider the non-linearity of the magnetic materials and iterative method [3,4] should be employed. The presence of sinusoidal sources leads to a time variation of the stored magnetic energy. In order to compute the magnetic energy accurately using only the first harmonic, and equivalent B(H) curve should be used [5].

In the case of voltage excitation, the winding current is unknown. By coupling the above Boundary Element Method with the circuit equation, the problem can be solved using an iterative method [6] to obtain the winding current, or by coupling the circuit equation to the BEM formulation [7].

Implementation

In a commercial software, OERSTED [8], the above boundary integral equation has been solved using the Galerkin method. Boundaries, i.e. the interfaces of materials, are modelled exactly by using segments (lines, arcs, or splines). These segments are then divided into small sections which are referred to as boundary elements. The equivalent surface current distribution on each element is approximated by linear shape function. When these discretized currents are used in (9) and (10) and the resulting equations tested with the same shape function on each element, a set of linear simultaneous equations for the unknowns of the surface equivalent current density coefficients is obtained. The system of equations is then solved for the coefficients of the expansion function. Once the equivalent currents are determined, the potential, EM fields, and induced currents can be calculated at any point by integration of the equivalent currents with the Green's functions. As well, parameters like torques, ohmic loss, and stored energy are readily calculated.

Using OERSTED on a PC or workstation, the geometry, material properties (conductivity, permeability, B(H) curve for nonlinear materials), excitations, and operating frequency can be entered by using a keyboard and a mouse/digitizer through a menu-driven interface. They will be displayed immediately. This approach minimizes human error and the time required to enter or modify a particular problem. The user may solve the problem interactively, or use the BATCH or parametric functions for unattended operation of the program. After the problem is solved, field value can be obtained at any desired location throughout the entire problem domain. Field distributions can be displayed in the form of contour plots, color maps, surface presentations, and graphs. Transverse field components can also be displayed in the form of arrow plots. The results can be saved in a data base for future use, and can also be printed out on hard copy.

Results

A 60 Hz three-phase, two poles, squirrel cage induction motor as shown in Figure 1 has been analyzed using OERSTED. The two-layer three-phase stator windings are indicated by letters a, b, and c. the prime on the letters indicates the reversed current direction. As previously mentioned, the analysis is performed from the viewpoint of a rotor observer. Thus we model the system that is excited by the current with slip frequency. Figures 1a and 1b show the real and imaginary parts of the flux, respectively, at slip frequency f=.3 Hz. Figure 2 shows a torque vs. slip frequency curve. We obtained this curve by using the parametric feature in OERSTED. We entered the geometry, the current source, the boundary elements, and the material properties. Then we defined the slip frequency as a parametric variable. The program automatically solved the problem at different frequencies. A total of 530 elements were used in the modelling; Figure 3 illustrates the element distribution. In this figure, the small section between two circular dots is an element. It should be noted that, due to the symmetry of the problem, only one pole pitch of the motor needs to be analyzed.

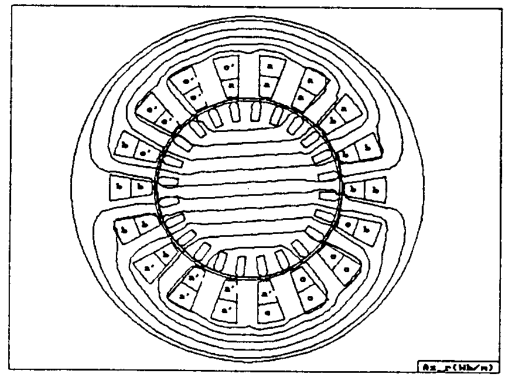

Figure 1 (a): Real part of the flux at f=.3 Hz.

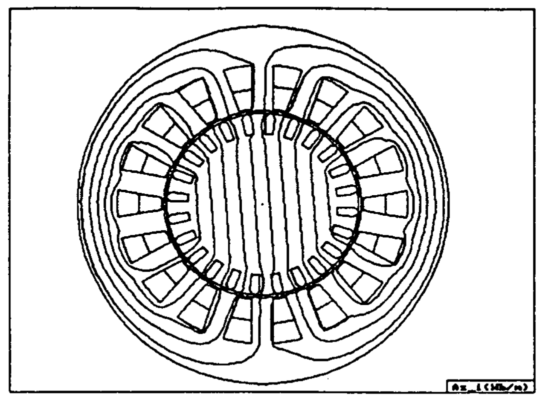

Figure 1 (b): Imaginary part of the flux at f=.3 Hz.

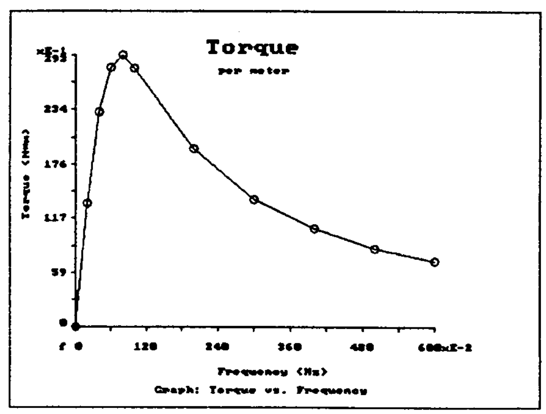

Figure 2: Torque vs. frequency curve for the motor in Figure 1.

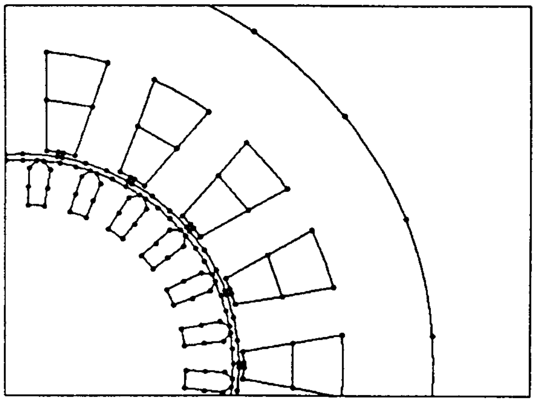

Figure 3: Element distribution.

Conclusion

A frequency domain Boundary Element Method has been presented for the analysis of polyphase induction machines. The analysis is performed from the viewpoint of a rotor observer. Therefore, the problem becomes a regular time-harmonic eddy current problem. An example is presented to show the advantages of the BEM technique, such as ease of use, accurate solutions, and a significant reduction of computation time.

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