A BOUNDARY ELEMENT ANALYSIS OF TEAM PROBLEM NO. 20: STATIC FORCE CALCULATION

ABSTRACT

Team problem 2D is specially challenging to many field theory codes in that it seeks to determine the force on a highly saturated rod which is centered inside a magnetic yoke. The magnetic fields near the edges of this rod are nearly singular in nature: any attempt to predict the forces using the magnetic fields around the rod, for example, with Maxwell stress tensor, would most likely meet with failure. Most finite element codes approach the calculation of force using the Coulomb energy technique [1], wherein the necessary derivatives on the magnetic field energy are taken inside the variational integrals. The boundary element approach described in this paper seeks yet another alternative for calculating the force. This boundary element approach attempts to represent the field both in the air and in the iron regions with a sheet of current on the air iron interface, with additional volume currents distributed throughout the medium which is saturated. Once the currents are determined, the magnetic field is computed using Biot-Savart and the forces are computed by summing the J cross B contributions.

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A BOUNDARY ELEMENT ANALYSIS OF TEAM PROBLEM NO. 20: STATIC FORCE CALCULATION

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INTRODUCTION

Problem #20 has been proposed by Professors Nakata, Takahashi and Fujiwara [2],[3]. Among the key features of the model are the following: the flux is three-dimensionally distributed, the ampere turns of the coil are more than sufficient to saturate the center rod and edges of the yoke, and the flux density changes rapidly along the perimeter of the central rod on which the force is being investigated. The fields have been accurately measured near the corners of this upper rod. Therefore, it is both the forces and the fields which are desired. Because the problem possesses quarter plane symmetry, some advantage in reduction of number of unknowns can be realized.

Figure 1 shows the problem geometry. The small center pole is forced to carry the flux for both yokes and is easily pushed into saturation above 2000 amp-turns excitation. The B field is requested along the lines a-b, c-d, Ψ-∃, and $\gamma - \dot{o}$.

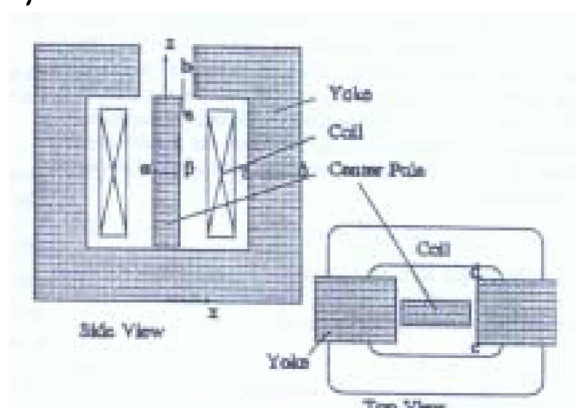


Figure 1 Geometry of Problem 20 showing the lines a-b,c-d, etc where fields are computed.

THEORETICAL DEVELOPMENT

As suggested in the abstract, the objective of this boundary element approach is to place unknown surface current on all air iron interfaces and seek to match the boundary conditions that normal B and tangential H be continuous across all such interfaces. Once found, all magnetic fields are determined by integration using the Biot-Savart law. Forces are also found using the integration of $\vec{J} x \vec{B}$ contributions. Because no derivatives are necessary with such a formulation, numerical errors are minimized. Because the medium (in this case, the center yoke) is highly saturated, additional volume currents are necessary to account for magnetization effects inside the iron. The contribution to the magnetic field and forces made by these volume currents is also accounted for by integration, thus reducing errors from differentiation.

Consider first the boundary element formulation for a non-saturable problem. It is wise to choose a formulation where the normal B field is guaranteed to be continuous. Let the magnetic vector potential A be defined as

$$\vec{A}(r) = \mu_0 \oint_s \vec{K}(r')G(r,r')ds'; G(r,r') = \frac{1}{4\pi\left|\vec{r}-\vec{r}'\right|} \tag{1}$$

The surface current K is laced over the air iron interface and has no normal component. In all of space, both air and iron, the B field can be represented as

$$\vec{B}(r) = \mu_0 \oint_s \nabla x \vec{K}(r')G(r,r')ds + \vec{B}_0 \tag{2}$$

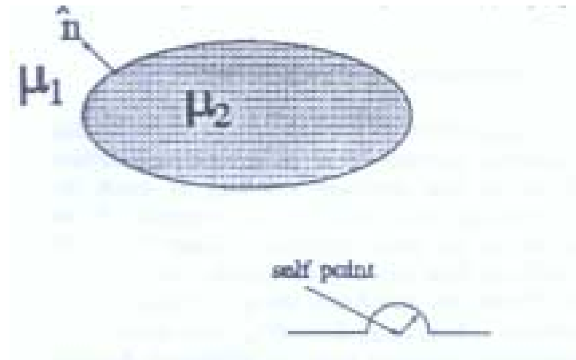
The field B_0 represents the impressed magnetic field from all other sources. In this case, B_0 is found using Biot-Savart with the coil that is wrapped around the yoke. Note that the unprimed curl operator operates only on the Green's function, but the surface gives the direction operator for the curl. It should be clear that (2) automatically enforces the condition that the normal B field be continuous. Only the condition on tangential H field need be enforced. Recall that the Cauchy principal part of (2) is

$$\int K \frac{\partial G}{\partial n} ds = \pm \frac{K}{2} \tag{3}$$

The plus or minus sign will depend on which side of the interface the normal is pointing towards. Equation (3) is determined by examining the integral,

$$\lim_{r-0} \int_0^{2x} \int_0^{\frac{x}{2}} \frac{1}{4\pi R^2} \sin\theta d\theta d\phi = \frac{1}{2} \tag{4}$$

which represents the integration around a hemisphere in the limit that the radius of the hemisphere goes to 0. It should be clear that the sign is positive if the normal is directed out of the region. By using the definitions of permeability, μ_1, μ_2 and the normal on the inset of Figure 2, it is possible to write



Integration when self point is on the surface

the expressions for the tangential field both in regions 1 and 2 as

$$nx\vec{H}_{t1} = \frac{\mu_0}{\mu_1} \int \hat{n}x\vec{K} \frac{\partial G}{\partial n}(r,r')dS' - \frac{\mu_0}{2\mu_1}\hat{n}x\vec{K} + \frac{\hat{n}x\vec{B}_0}{\mu_1} \tag{5}$$

$$nx\vec{H}_{t2} = \frac{\mu_0}{\mu_2} \int \hat{n}x\vec{K} \frac{\partial G}{\partial n}(r,r')dS' + \frac{\mu_0}{2\mu_2}\hat{n}x\vec{K} + \frac{\hat{n}x\vec{B}_0}{\mu_2} \tag{6}$$

Equating (5) and (6) yields a single integral equation for the unknown fictitious surface current K as

$$\oint \hat{n}x\vec{K}(r') \frac{\partial G(r,r')}{\partial n} ds' + \frac{\hat{n}x\vec{K}}{2}\left(\frac{\mu_1+\mu_2}{\mu_1-\mu_2}\right) = -\frac{\hat{n}x\vec{B}_0}{\mu_0}. \tag{7}$$

It is useful to consider the physical significance of this fictitious surface current K that has been developed. In the real world, H can only be discontinuous if a surface current exists; this boundary condition is written $\hat{n}x\left|\vec{H}\right| = \vec{K}_f$. By analogy it should be clear that this fictitious surface current K represents a discontinuity in the magnetization or

$$\hat{n}x\left|\vec{M}\right| = \vec{K}. \tag{8}$$

In any event, once the surface currents are determined, the B field can be computed in post processing easily using

$$\vec{B} = \mu_0 \int \vec{K}(r') \frac{\partial}{\partial n} G(r,r')ds' + \vec{B}_0 = \vec{B}_{ind} + \vec{B}_0. \tag{9}$$

Note that the permeability multiplying the integral in (9) is that of free space.

Because the problem is nonlinear, it is necessary to add volume currents in all saturable regions. Actually, these volume currents are needed whenever $\nabla x\overrightarrow{M} \neq 0$. Defining a magnetization current $\vec{J}_m \equiv \nabla x\overrightarrow{M}$ it is possible to obtain a modified expression for the computation of the B field as

$$\vec{B} = \vec{B}_0 + \oint_{\Gamma} \nabla x(K(r')G(r,r'))ds' + \int_v \nabla x(\vec{J}_m(r')G(r,r'))dV \tag{10}$$

Obviously these volume currents, J_m, are unknown a priori. Solution of the problem is as follows: First, assume $J_m = 0$; second, compute the B field everywhere; third, check all saturable regions to determine whether $\nabla x\overrightarrow{M} \neq 0$; and substitute $\vec{J}_m = \nabla x\overrightarrow{M}$. The process is then repeated until no change in these magnetization currents is witnessed. Under relaxation can be utilized to accelerate conversion, with an under relaxation parameter of about 0.3. Throughout this process the magnetization current is always a source term and is treated as known. It is computed after the fact and forms a contribution to the right-hand side of the equations being solved.

The question immediately arises as to the best method for computing the magnetization current, J_m. It is instructive to consider first how not to do this. The most straightforward way might perhaps be to use equation (10) to compute the magnetic field throughout the saturated region. In any local region, the B field would be known at 6 points and 3 orthogonal directions. It is easy to extract the magnetization M from the B-H curve for each of these 6 points and perform a finite difference approximation to the curl operator for the determination of J_m. The amount of error associated with this approach is usually quite high. A better approach is to begin with the definition of magnetization as

$$\vec{M} = \frac{\vec{B}}{\mu_0} - \vec{H} = \vec{B}\left(\frac{1}{\mu_0} - \frac{1}{\mu}\right) = \vec{H}\left(\frac{\mu}{\mu_0} - 1\right) \tag{11}$$

The magnetization current would then follow from the curl of this expression as

$$\vec{J}_m = \left(\frac{\mu}{\mu_0} - 1\right)\nabla x\vec{H} = \frac{1}{\mu_0}\nabla\mu x\vec{H}\ . \tag{12}$$

The curl of H would be the actual current imposed within the steel region; it happens to be equal to 0 for this particular problem. If it were not 0, however, one would begin by simply multiplying the actual impressed current by the relative permeability since the total volume current in the region would be equal to J_m plus the real impressed current. This is perhaps made clear by expanding the terms in (12) as

$$\vec{J}_m = (\mu_r - 1)\vec{J}_{real} + \frac{1}{\mu_0}\frac{\partial\mu}{\partial|B|}(\nabla|\vec{B}|x\vec{H})\ . \tag{13}$$

Adding the first term in (13) to the real impressed current equates to simply multiplying the original impressed current by the relative permeability. It is the second term in (13) that is more difficult to handle. By explicitly writing the gradient of permeability in terms of $\frac{\partial\mu}{\partial|B|}$, it is possible to extract the information necessary for the computation of J_m directly from the kernels of the integral equation. The term $\frac{\partial\mu}{\partial|B|}$ follows directly from the B-H curve for the material. The H field needed in (13) follows by dividing (10) by the local permeability μ, also determined from the B-H curve. The gradient term is computed as

$$\nabla|\vec{B}| = \mu_0 \int_r \vec{K}(r')\nabla|\nabla xG(r,r')|dS' + \mu_0 \int_v \vec{J}_m(r')\nabla|(\nabla xG(r,r')|dV' + \nabla|\vec{B}_0| \tag{14}$$

Thus, the magnetization current is determined at all the corners of a subvolume within the magnetization region, and interpolation used within the subvolume to distribute this magnetization current. It should be further noted that each of the Green's functions in (14) take on the direction of the surface current K and magnetization J, respectively.

A picture of the subvolumes used in this problem is shown in Figure 3. A total of 1,943 were used, 800 alone for the center pole with 200 crowded into the upper section of the center pole. Starting with the assignment, J_m=0 everywhere, all the surface currents and the magnetic fields associated with them are determined at the corners of these subvolumes. After the first iteration the magnetization current is computed from (13). The process is then repeated. In the second iteration, the surface currents are again the unknowns; the volume currents are assumed to be those dictated by the first iteration. Once the new surface currents are found in the second iteration, magnetic fields are

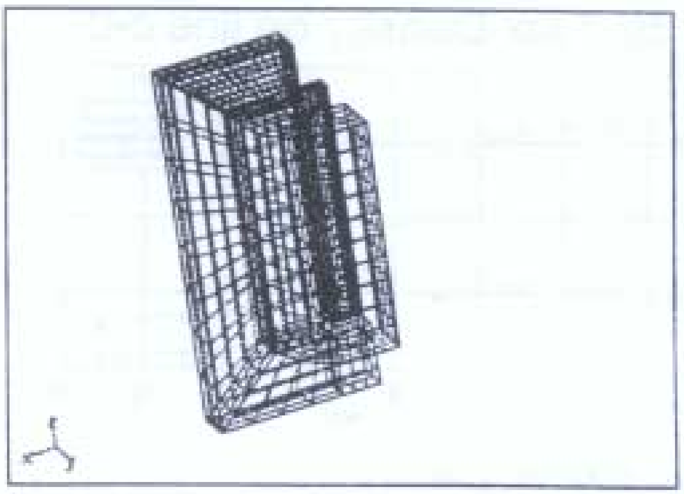


Figure 3 Subvolumes used for the problem.

determined throughout the saturable regions and new magnetization currents are computed. They are updated from the second iteration using the relaxation formula

$$J_m^{k+1} = \beta J_m^{k+1} = (1-\beta) J_m^k . \tag{15}$$

For this problem $\beta = 0.2$. The equations are solved for the unknown surface current at each iteration using an iterative conjugate gradient solver. Convergence criteria for each iteration was set to terminate with changes less than 0.3% changes. Since quarter plane symmetry was assumed, the number of unknowns was only 1,200. The problem was worked on an HP Workstation 710 having a 12.2 megaflop speed rate, and was solved in roughly 10 hours.

RESULTS

The fields and forces were required for 4 different current

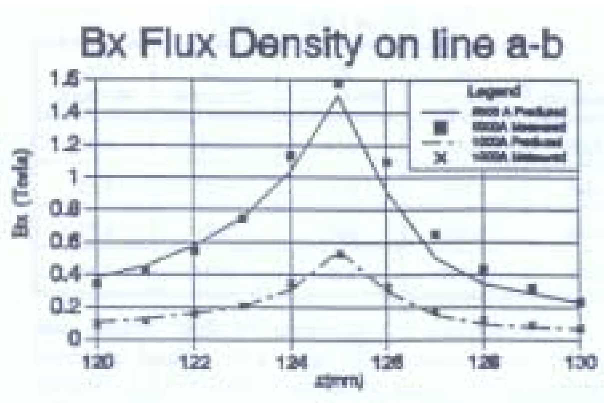


Figure 4 Field predictions on the line a-b.

settings of 1,000 – 5,000 amp turns. Shown in Figure 4 is the field prediction on line a-b shown compared to the measured data. The predicted fields are slightly lower (=10%) than the measured data.

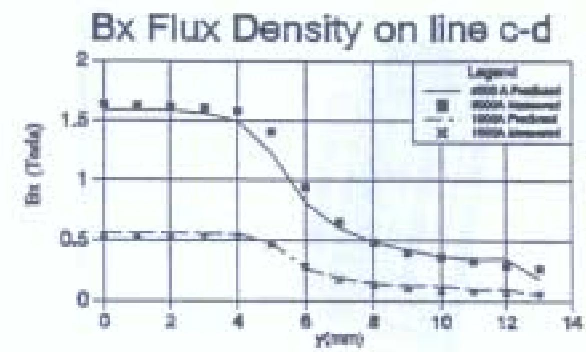


Figure 5 Bx on line c-d.

By comparison, the field comparisons on line c-d are shown in Figure 5. Again the computed fields are lower than the measured data.

In spite of this lower field prediction, the forces were quite good. The forces are summarized in Table I.

Table I Fz force Calculations

Amps (AT)	Force (Nt) Computed	F measured
1000	7.61	8.1
3000	52.41	54.4
4500	73.65	75.0
5000	79.38	80.1

The points P1 and P2 are located at the (x,y.z) locations (0,0,25.75) and (12.5, 5, 25.75) respectively. The results are summarized in Table II.

Table II Bz at points P1 and P2

Amp-Turns(At)	P1 Computed	P1 Measured	P2 Computed	P2 Measured
1000	0.303	0.36	0.242	0.24
3000	0.797	0.84	0.613	0.63
4500	0.955	0.99	0.694	0.72
5000	0.993	1.03	0.714	0.74

The average z component of field density was computed along lines $\alpha - \beta$ and $\gamma - \dot{o}$. These positions corresponding to (x,y,x) locations (-12.5<x<12.5, -5<y<5, z=75) and (38.5<x<63.5, -12.5<y<12.5,z=75) respectively. The results are summarized in Table III.

Table III Bz at points P1 and P2

Amp-Turns(At)	Alpha-beta Computed	Alpha-Beta Measured	gamma-delta Computed	Gamma-delta Measured
1000	0.713	0.72	0.132	0.13
3000	1.79	1.75	0.357	0.36
4500	2.01	2.01	0.44	0.43
5000	2.06	2.05	0.462	0.46

CONCLUSIONS

The boundary element method is, indeed, an accurate option for solving nonlinear highly saturable magnetic field problems such as this. As the degree of saturation increasing as it did for the final current setting, the number of subvolumes necessary for accurate solution is quite high. In this domain it could be argued that the boundary element method is similar to a finite element approach in that the number of unknowns goes up as the problem density cued. However, the number of unknowns remains an n^2 type problem since it is only the surface currents that are determined from a full matrix solution at every iteration. In practice it is found that the magnitude of the magnetization currents is considerably smaller than the surface currents K that were being predicted. Indeed, unless the problem isn't driven considerably into saturation, reasonably accurate solutions of 10% or less can be had without any subvolumes. One nice feature of the boundary element method certainly the fact that forces are computed by summing up J cross B terms throughout the problem. Summations like this circumvent difficulties of other approaches requiring derivative of energy.

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