

THREE DIMENSIONAL ANALYSIS OF MAGNETIC FIELDS USING THE BOUNDARY ELEMENT METHOD

ABSTRACT

Increasingly complex electromagnetic devices require more flexible modelling, and fast and accurate computation of performance parameters. Although two-dimensional analysis provides good approximations, often the physics of the problem requires three-dimensional analysis. Using the Boundary Element Method (BEM) for three-dimensional analysis, now provides unprecedented design opportunities to engineers. This paper outlines the basis of three-dimensional BEM design and analysis of magnetic devices, and contrasts 2D/3D design for optimal utilization of 3D. Options to test solution accuracy are discussed. The torque converter sample problem demonstrates the interactive graphics capabilities, the efficiency and power of BEM in the analysis of magnetic fields.

Integrated Engineering Software - Website Links

[Home](#)[Products](#)[Support](#)[Technical Papers](#)

"Page Down" or use scroll bars to read the article



THREE DIMENSIONAL ANALYSIS OF MAGNETIC FIELDS USING THE BOUNDARY ELEMENT METHOD

Y. Bulent Yildir, Bruce W. Klimpke, K. M. Prasad

Integrated Engineering Software Inc.
347-435 Ellice Avenue, Winnipeg, Manitoba R3B 1Y6

Increasingly complex electromagnetic devices require more flexible modelling, and fast and accurate computation of performance parameters. Although two-dimensional analysis provides good approximations, often the physics of the problem requires three-dimensional analysis. Using the Boundary Element Method (BEM) for three-dimensional analysis, now provides unprecedented design opportunities to engineers. This paper outlines the basis of three-dimensional BEM design and analysis of magnetic devices, and contrasts 2D/3D design for optimal utilization of 3D. Options to test solution accuracy are discussed. The torque converter sample problem demonstrates the interactive graphics capabilities, the efficiency and power of BEM in the analysis of magnetic fields.

INTRODUCTION

Two distinct approaches exist for the solution of boundary value problems: domain-type and boundary-type.

Domain-type formulation is the direct solution of the differential equation governing the field. The finite difference (FDM) and finite element (FEM) methods are the two most commonly used direct methods.

In the finite difference approach, the differential operator is discretized, utilizing a truncated Taylor series expansion in each coordinate direction, and applied at each point of a rectilinear grid placed on the problem region. The method usually involves an iterative process.

The finite element method uses a variational technique in which the potential is approximated by a sequence of functions defined over the entire domain of the problem. Relating these approximations to the operator equation through minimizing a functional that is proportional to the energy of the system yields the nodal values of the solution.

The second approach to the solution of boundary value problems is the boundary integral equation formulation on which the Boundary Element Method (BEM) is based.

For over 20 years, BEM CAE technology has been used extensively in civil, mechanical and nuclear engineering. A major area in which it has been employed is the CAE design and analysis of microstips, antennas and propagation scatterers [2].

Until recently, largely due to historical reasons and relative simplicity of implementation, FEM received the most research attention. The major difficulty in developing BEM packages for electrical engineering lay, not in the basic theory, but in translating its complexity into programs easily used by the design engineer [3,6].

In the integral equation formulation the potential is not solved for directly, but an equivalent source, which would sustain the field, is found by forcing it to satisfy prescribed boundary and interface conditions under a function which relates the location and effect of the source to any point on the boundary [4].

This function, called the Green function or influence function, effectively eliminates the need of a finite element mesh or a finite difference grid.

The main advantages of this method over the direct approach are the reduction of one in problem dimensionality, accurate modelling of geometry, elimination of differentiation and interpolation to calculate potential or its derivatives, precise results due to the smoothness of the integral operator and sound means for checking the accuracy of the solution [5, 7].

BOUNDARY ELEMENT METHOD

To solve the magnetostatic field equations, in integral form, the boundary element method is applied. For linear problems the unknowns are only required on the surfaces where the permeability on either side is different, or on surfaces of permanent magnets. The number of unknowns is thus relatively small as compared to domain methods (such as finite difference or finite elements) where the entire volume must be discretized.

The approach taken is to approximate the permeable materials and permanent magnets by an equivalent surface current. To accomplish this, the surface currents must be approximated by a set of basis functions. The coefficients of the basis functions are then determined by the weighted residual method which enforces the boundary conditions on \mathbf{B} at the interface.

Linear basis functions were chosen to model the equivalent current; however, others could have been used. Upon discretization we wish to construct a system of equations

$$\mathbf{S} \mathbf{K}' = \mathbf{J} \quad (1)$$

where \mathbf{K}' is a vector of unknown coefficients, \mathbf{J} is the vector of source terms, and \mathbf{S} is the system matrix. The elements of the \mathbf{S} matrix and \mathbf{J} vector were calculated by Galerkin's method [5].

The system of equation is then solved for the coefficients of the expansion functions. Once the equivalent currents are determined, the potential or the magnetic field can be calculated at any point by integration of the equivalent currents with the Green function. As well, parameters such as forces, torques and inductances are readily calculated.

For nonlinear problems, the permeability is a function of the \mathbf{B} field. Thus the problem must be solved repeatedly until convergence in material properties has been achieved [9]. In addition, the nonlinear materials may have to be subdivide into subvolumes where an equivalent volume magnetization is calculated.

$$\mathbf{J}_m = \nabla \times \mathbf{M} \quad (2)$$

The unknown volume currents are not calculated as part of the system matrix but are iterated on until a stable value is reached and the result put in the right hand side of (1). Thus the equivalent volume magnetization occurs as unknowns in the right hand side rather than in the system matrix, as is the case with volume integral formulations. For many problems the effect of the volume magnetization is small and can be neglected. Practically the effect of volume magnetization is only noticeable in volumes where the field is rapidly changing from a saturated to unsaturated state.

Subvolumes often have a small effect on the total field and in such cases they can be neglected.

From a practical viewpoint the most attractive feature of the BEM is that open region problems are catered to inherently [8]. When using domain methods one must attempt to generate three-dimensional meshes to some arbitrary distance where a boundary condition is assumed. For many applications this can in practice be impossible.

MAGNETIC FIELD EQUATIONS

For the magnetostatic field we seek the solution of the vector potential due to source currents such that:

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (3)$$

The magnetic vector potential due to an arbitrarily oriented surface current density is

$$\mathbf{A}'(r) = \frac{\mu_0}{4\pi} \int_s G(r, r') \mathbf{J}(r') ds' \quad (4)$$

The calculation of the magnetic vector potential in free space requires the integration over all current sources.

If regions of different permeability are present, their effects can be accounted for by an equivalent current or magnetization [1]. That is, the vector potential due to a homogeneous magnetized body is identical to the surface currents whose density is

$$\mathbf{K} = \mathbf{M} \times \mathbf{n} \quad (5)$$

The contribution to the magnetic vector potential due to an equivalent magnetization current on the boundary is:

$$\mathbf{A}^K(r) = \frac{\mu_0}{4\pi} \int_R G(r, r') \mathbf{K}(r') dr' \quad (6)$$

To determine \mathbf{K} , the interface equation for the continuity of the tangential component of the field intensity is enforced

$$n\alpha(H_2 - H_1) = 0 \quad (7)$$

or

$$n\chi \left(\frac{B_2}{\mu_2} - \frac{B_1}{\mu_1} \right) = 0 \quad (8)$$

Substituting (3) into (8) for \mathbf{B} yields a second kind Fredholm integral equation in terms of \mathbf{K} . This equation can be solved for \mathbf{K} and subsequently \mathbf{A} can be calculated anywhere by addition of (4) and (6).

SOLUTION ACCURACY and VERIFICATION

The accuracy of the solution using the BEM depends upon a number of factors. The three most important factors are how well the integrations are performed, the number of boundary elements used and the placement of the elements. The first is dependent only on how well the programmer of the software can achieve a desired accuracy of integration within a specified computation time. By taking longer to perform integrations fewer elements may be required for some problems. As techniques evolve to achieve greater integration accuracy within a specified time, this problem is expected to become inconsequential.

The location and number of boundary elements is currently the responsibility of the person using the software. More elements are required on surfaces in close proximity to other surfaces. As well, the more rapid the field variation the more elements are required.

To test if the element distribution is adequate some basic checks can be applied. The simplest is increasing the element density and checking if the solution changes significantly. Significant changes indicate that more elements may be required. Ampere's circuital law can be applied over a variety of closed paths to verify the solution as well.

Future work will be directed at finding surface locations where element density is not adequate. Improvements to the element density can subsequently be made by the user or eventually by the software itself.

MICROCOMPUTER IMPLEMENTATION

The 640kb barrier on RAM has been limiting the use of PC-type micro computers for implementing 3-D electromagnetic software. Now, with the extended memory feature in micros, the size of the software used can be practically as large as the user wishes. In addition to large memory requirement, computation speed is equally important to 3-D field problems. The new micros, with special math co-processors, offer speed comparable to that of a mainframe. Therefore, the sophistication of solving a 3-D problem is limited only by the scope of the software used.

A 3-D magnetic field problem is described by defining the geometry, material properties, and sources. Identification of the direction of the sources in 2-D is trivial, whereas in 3-D, an intelligent algorithm is essential to reduce the volume of input data required. Also, displaying 3-D objects on a 2-D screen requires special expertise.

The simulation of fields on micro computers require the input of the above, the numerical solution of the field equation, and output of the desired parameters. The process is repeated until optimum values for the design parameters are obtained. The efficiency of the procedure is measured by the amount of time required to complete the design. The factors that affect the efficiency are the ease of use, the accuracy of results and the capabilities and speed of the program. Of all these factors, perhaps ease of use is the most significant because of the complexity of viewing/handling the parameters of a 3-D problem. In a boundary element package discretizing media interfaces is sufficient, unlike the need of a 3-D mesh in the entire region of the problem for a finite element analysis. This is by far the biggest simplification and hence the time saver.

Ease of use is not a concept totally isolated from the numerical technique used. There are a number of important factors contributing to the ease of use which are brought about by the solution technique used in the package.

The user interface in AMPERES was designed to require minimal keyboard entry and hand motion. Menus are structured to follow the natural pattern of defining and solving a problem and to incorporate the same sets of commands that operate on different objects. On-line help is provided in every menu.

A great majority of problem defining parameters can be entered with the help of a mouse and immediately displayed. This approach minimizes human efforts and error, and the time required to modify a particular problem.

The special features of the micro computer environment (eg. fast colour graphics, colour printer, mouse or keyboard entry, math co-processor, hard disk, RAM disk) are fully utilized to integrate problem definition, analysis, data storage, drafting and presentation capabilities.

A SAMPLE ANALYSIS

In this section, a sample problem is presented using AMPERES on an IBM PC-type micro computer. The results obtained are compared with the measured value.

The example is the magnetic clutch illustrated in Fig. 1. The device consists of two distinctly separated parts – rotor and stator. A steel cylindrical core with 10 permanent magnets attached to it forms the rotor.

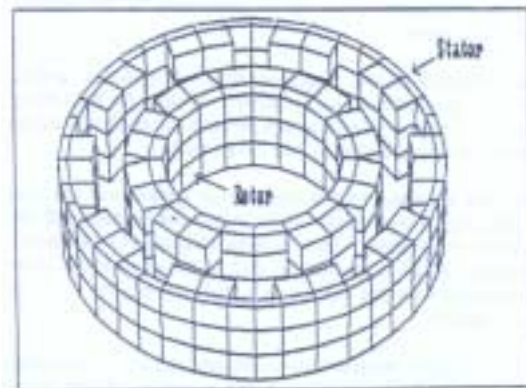


Fig.1: Magnetic clutch geometry.

Stator is also constituted by 10 magnets internal to a steel core. Dimensions of the device and the directions of the permanent magnets are shown in Fig. 2

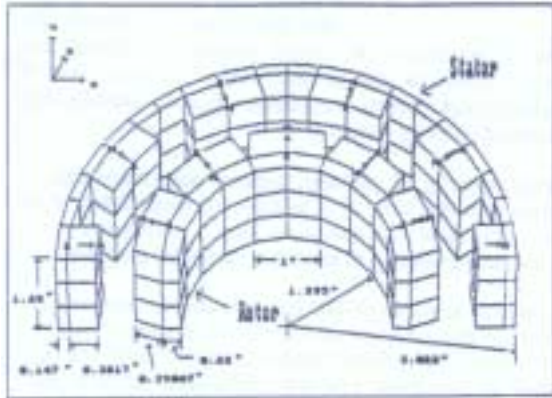


Fig.2: Dimensions and magnet orientations.

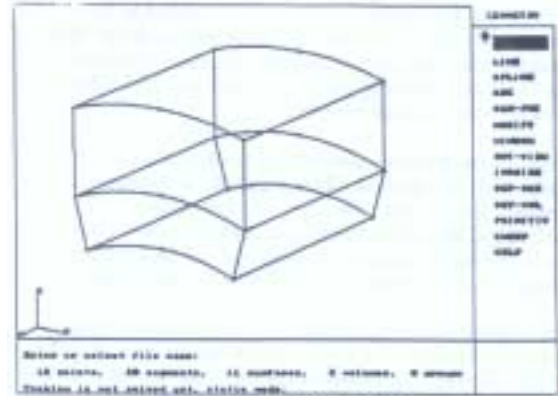


Fig.3: Primary rotor section.

Often 3-D objects displayed on a 2D screen deceive the user's vision. This inconvenience can be minimized in AMPERES by modelling the problem in parts, and then merging them to obtain the total problem. Therefore, in the present example, rotor and stator are modeled separately, and merged later.

The problem is modeled by entering the geometry, nonlinear B-H curves for the steel and permanent magnets. Boundary elements are then discretely placed on the surfaces where they are required. If a problem consists of volume currents, then sub-volumes have to be generated in the current carrying volumes.

The geometry of the magnetic clutch is given in inches. This can be chosen to be the system of units for data entry and result output. The program permits selection of various systems of units, including the user's own.

After the selection of the proper unit system and the real number range in cartesian coordinates, the geometry of the problem is entered. This is done through the geometric modeller, which is an integral part of the program. Either the keyboard or the mouse or a digitizer can be used for this purpose.

The geometric modeller provides commands for creating, modifying and inquiring about the geometry. Using these commands, 3 types of geometric items – segments, surfaces and volumes can be generated. Surfaces are defined by either 3 or 4 closely connected segments and volumes by surfaces. However, boundary elements can be generated only on 4 sided surfaces, and sub-volumes only in 6 surfaced volumes.

First a primary 36° section of rotor is modeled, as shown in Fig. 3. This is best achieved in a small window. Then the rotor in full is generated by rotating and duplicating the primary block. Before creating the complete rotor assembly, the material properties, boundary elements and magnetization direction are assigned to the primary rotor block.

The type of steel used for the rotor and stator cores is "MalFe", and the permanent magnet material is "R5". The normalized B-H curves, provided by the manufacturer, of these two materials have to be generated first for defining the material properties. AMPERES is supplied with a default table of 13 materials, as shown in Fig. 4.

Material	Density	Poisson
STEEL	0.0000	0.3000
ALUMIN	0.0000	0.3000
COPPER	0.0000	0.3000
GLASS	0.0000	0.2000
RUBBER	0.0000	0.4000
OTHER	0.0000	0.3000

Fig.4: Default material table.

Each non-linear material is associated with a default B-H curve, which can be modified by the user. In AMPERES, 2 to 20 points define a normalized b-H curve.

For the "MalFe" material 12 points were extracted from the manufacturer's curve. The points in the form (H(Oersted), B(Tesla)) are as follow: (0,0), (2,0.3), (4,0.85), (8,1.25), (10,1.351), (20,1.502), (40,1.654), (80,1.758), (199.8,1.89), (1508,2.19), (4901,2.575), and (4995,2.58). The curve then appears as in Fig. 5.

Magnets are defined by entering their second quadrant or demagnetization curve. The next step is to assign these materials to the volumes of the primary rotor block by selecting them from the menu and locating the cursor in the desire volume. Additional requirement for magnetic material regions is magnetization direction identification. Direction numbers (0,0,1) are assigned to the magnet in the primary block.

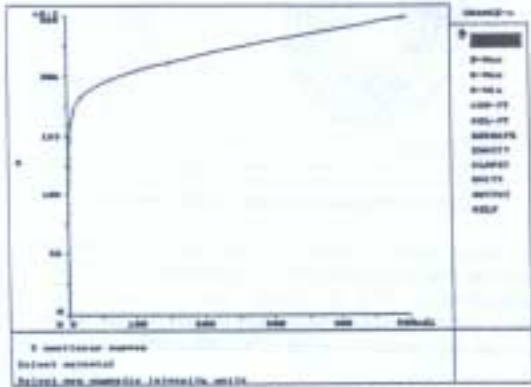


Fig.5: The non-linear B-H curve for "MalFe".

ATTRIBUTE TO BE COPIED	YES/NO
1. GEOMETRY	Y
2. MEDIA	Y
3. MESH	Y
4. MEDIUM NUMBER	Y
5. MEDIUM NAME	Y
6. MEDIUM PROPERTY	Y
7. MEDIUM DIRECTION	Y
8. MESH	Y

Fig.7: Copy-enable table for parameters.

The most crucial part to solving the problem accurately is the generation of boundary elements. Boundary elements are generated only on surfaces carrying currents and surfaces between dissimilar media, or magnets with dissimilar magnetization. The element configuration used for the primary rotor block is shown in Fig. 6

After setting the copy enable flags of elements, media to "y(yes)" the primary rotor block is rotated by 36° and duplicated as shown in Fig. 8.

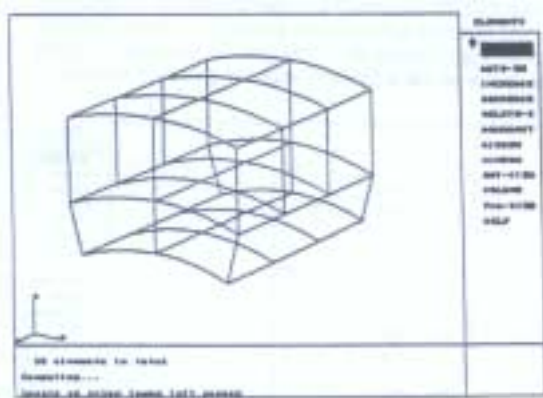


Fig.6: Element structure on primary rotor block.

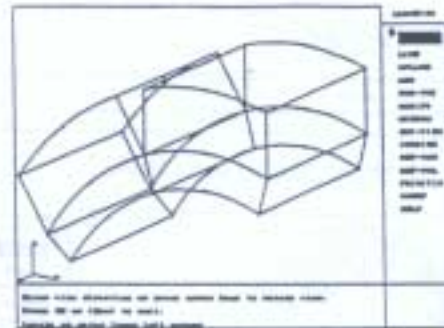


Fig.8: Geometry after rotating the primary block by 36°.

Now the permanent magnet of the newly generated rotor section has to be assigned with direction numbers (0,sin36°,-cos36°). Creating 8 more similar sections at an interval of 36° completes the rotor assembly, shown in Fig 9.

Two core surfaces that are parallel to X-axis have not been discretized because they would become interfaces between similar media after rotating and duplicating the primary block.

In AMPERES a geometry can be duplicated with all its attributes including elements, media, magnet directions, currents, etc.. In Fig. 7, attributes that can be chosen to be copied along with the geometry are listed.

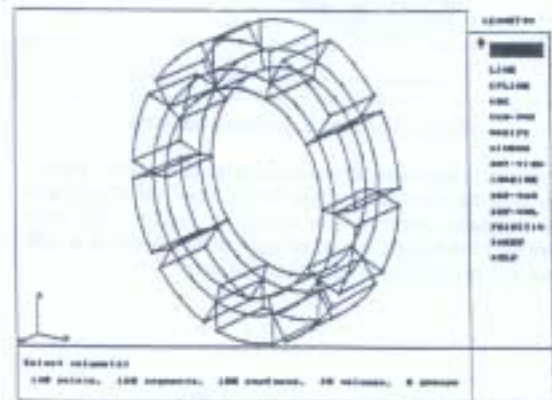


Fig.9: The complete rotor assembly.

Every time a new section is created, appropriate direction numbers have to be assigned to the magnet of the individual rotor section. This completed part of the device is saved in a database file called "rotor".

The stator is constructed using the same method. The primary stator block, shown in Fig. 10, is used to obtain the complete stator assembly shown in Fig. 11.

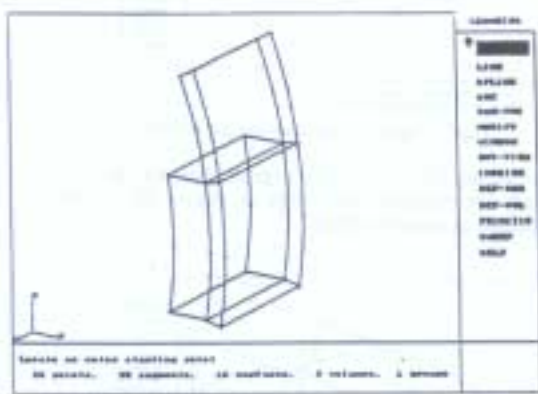


Fig.10: The primary stator block.

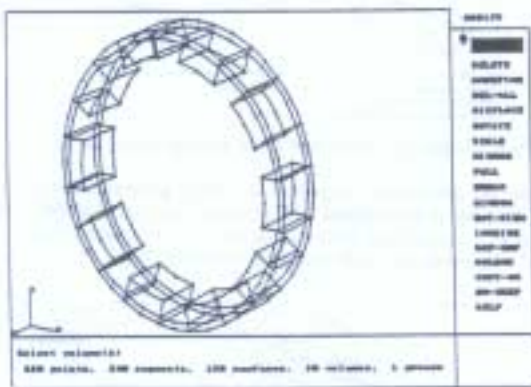


Fig.11: The complete stator assembly.

The stator information is saved to a file called "stator". Merging the two database files "rotor" and "stator" produces the magnetic clutch device shown in Fig. 12. The total number of boundary elements used in this problem is 800. Now the problem is ready to be solved.

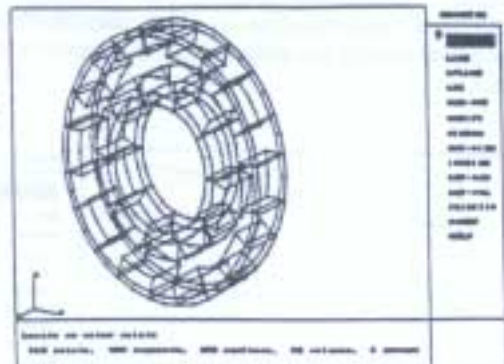


Fig.12: The complete magnetic clutch assembly.

After the solution is obtained, various parameters can be calculated. The torque about the axis of the device has

been calculated to be 177.07 lbs x inch. The measured torque value is 162 lbs x inch. The difference between the calculated and the measured values is less than 10%.

At first glance, one may think that a 2-D analysis of this problem could provide reasonable results. To verify this hypothesis, the same problem was solved by our 2-D software MAGNETO which is also based on the Boundary Element Method. The torque obtained with MAGNETO for the device height (1.25 inch) is 317 lbs x inch, which is almost twice the measured value. Clearly, a 3-D analysis is required to accurately model the problem.

In AMPERES an arrow plot of the B-field can be calculated on any selected surface of the geometry or on any arbitrary plane. Fig. 13 shows the arrow plot on the plane bisecting the device in the x-direction with the geometry masked.

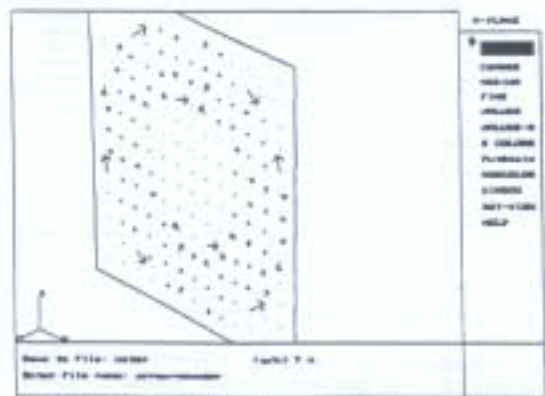


Fig.13: Arrow plot of magnetic induction (B) on a plane.

Graphs of vector magnetic potential and B-field can be obtained along any arbitrary line or curved segment or along a curve generated between any two points on a selected surface. The other important feature in AMPERES is the calculation of constant potential/field contours on multiple selected surfaces or on any arbitrary plane. Any other information for different rotor position can be obtained quite efficiently by using the batch option. Many what-if cases can be analyzed unattended.

CONCLUSIONS

The boundary element technique, applied to magnetic field problems, has some unique advantages as compared to more traditional domain methods such as finite element or finite difference techniques. A major advantage of the BEM is that open region problems are handled without artificially truncating the problem. As well, the integral formulation is used and unknowns are only required on the surface, except where nonlinear materials are rapidly changing from an unsaturated to saturated state. Subvolumes may be required in such cases to improve solution accuracy.

The number of unknowns required is relatively small. However, boundary element techniques typically generate large dense matrices. In order to solve these matrices iterative solvers with fast disk caching techniques are required. These techniques enable the solution of 3D problems on powerful personal computers such as the Intel 80486 line.

Calculations of the magnetic field, magnetic vector potential, forces, or torque requires integrating the equivalent unknowns with the appropriate Green function. Provided the integrations are handled properly, the resulting calculations are very accurate due to the smoothing effect of the integral operator.

Data input is inherently straightforward due to the reasons mentioned above; unknowns are located only on boundaries and there is no need to generate a volumetric mesh in the unbounded region.

For most practical problems, ease of the initial problem specification is secondary to subsequent modifications. For example, assume the torque is required for a motor with the rotor position varying with respect to the stator. After the solution is obtained for one position, the rotor can then be rotated to a new position without regenerating a new mesh. A finite element solution would require generating a new mesh in the air gap between the stator and rotor for each new position.

ACKNOWLEDGEMENTS

We acknowledge the invaluable contributions of our colleagues at Integrated Engineering Software Inc.

Development of the two-dimensional magnetic field analysis package was funded in part by an IRAP-M grant from the National Research Council of Canada.

REFERENCES

- [1] Stratton, J.A. *Electromagnetic Theory*. New York: McGraw-Hill Book Co., 1941.
- [2] Harrington, R.F. *Field Computation by Moment Methods*. Reprinted by R.F. Harrington, R.D.2, West Lake Road, Cazenovia, N. Y., 1968.
- [3] Mikhlin, S.G. *Integral Equations*. New York: Pergamon Press, 1964.
- [4] Jaswon, M.A. and Symm, G.T. *Integral Equation Methods In Potential Theory and Elastostatics*. New York: Academic Press, 1977.
- [5] Yildir, Y.B. *A Boundary Element Method for the Solution of Laplace's Equation in Three-Dimensional space*. Ph.D. Dissertation, University of Manitoba, 1985.
- [6] Lean, M.H. *Electromagnetic Field Solution with the Boundary Element Method*. Ph.D. Dissertation, University of Manitoba; 1981.
- [7] Klimpke, B.W. *A Two-dimensional Multi-media Boundary Element Method*. M.Sc. Dissertation, University of Manitoba, 1983
- [8] Deno, D.W. "Transmission Line Fields", *IEEE Trans. Power Apparatus and Systems*, Vol. PAS-95, No. 5, September/October 1976.
- [9] Lean, M.H. and Bloomberg, D.S. "Nonlinear boundary element method for two-dimensional fields", *Journal of Applied Physics*, Vol. 55, No. 6, March 1984.