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The less common method of solving magnetic field problems is using the boundary element method. It has been used successfully on many magnetic field problems including sensors, NMR machines, and beam analysis problems. Boundary elements are associated with solving Maxwell's equations in integral form. Both methods have some inherent advantages and disadvantages.

The ideal solution would be to solve a given problem where the advantages of both methods could be applied simultaneously. This paper discusses the advantages of such an approach.

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A Hybrid Magnetic Field Solver Using a Combined Finite Element/Boundary Element Field Solver

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__Abstract__ - The dominant method to solve magnetic field problems is the finite element method. It has been used successfully on many devices including motors, solenoids, and actuators. More formally using finite elements is a method of solving Maxwell's equations in differential form. The less common method of solving magnetic field problems is using the boundary element method. It has been used successfully on many magnetic field problems including sensors, NMR machines, and beam analysis problems. Boundary elements are associated with solving Maxwell's equations in integral form. Both methods have some inherent advantages and disadvantages. The ideal solution would be to solve a given problem where the advantages of both methods could be applied simultaneously. This paper discusses the advantages of such an approach.

INTRODUCTION

Two fundamentally different approaches can be used to solve Maxwell's equations. The most common method is to write the governing equation in differential form, such as Poisson's equation, which we will discuss later. The most used numerical technique used to solve equations posed in differential form is the Finite Element Method (FEM). The Finite Difference (FD) method can also be employed but has some fundamental limitations.

The less common method is to write the governing equation in integral form. Numerically the problem can then be solved by the Boundary Element Method (BEM). We will see how Poisson's equation can be posed in integral form and then solved.

Both of these methods have large advantages and disadvantages depending on the geometry, material properties and the required accuracy of the final solution. Thus, in general, it is advantageous to combine both differential and integral equation solvers to take advantage of there strengths to solve a given problem. Some real world problems are discussed to illustrate the advantages of both the finite element and boundary element numerical solvers and why a combination of both is preferred.

FEM METHOD

In general any field problem can be expressed in the form

$$L\phi = g$$

where ϕ is the exact solution, L is an operator, and g is the source or forcing function. For finite elements L is a differential operator and ϕ is the solution, which is normally a scalar or vector potential. The magnetic vector potential is governed by the differential equation

$$\nabla \times \frac{1}{\mu} \nabla \times \vec{A} = \vec{J}$$

where μ is the permeability of the material and $\vec{J}$ is the source current density. Assuming the permeability is linear this can be simplified to

$$\nabla \times \nabla \times \vec{A} = \mu\vec{J}$$

The operator is the curl-curl operator, $\vec{A}$ is the quantity to be determined and $\mu\vec{J}$ is the source function.

For all practical problems it is impossible to solve for $\vec{A}$ exactly. To arrive at an approximate solution we can use a Galerkin approach. Simply put, we define a inner product throughout the domain of the problem, the solution minimizing the energy in the system. That is we construct a linear system of equations derived from an inner product

$$\langle L\phi, \propto \rangle = \langle g, \propto \rangle$$

where $\propto$ is a set of basis functions defined over the domain of the problem.

For the curl-curl operator we will find the best approximation for $\vec{A}$ over the domain using the $\propto$ basis functions. That is

$$\vec{A} = \sum_{i=1}^{N} A_i \propto_i$$

where the $\propto_i$ are a set of scalar basis functions.

For simplicity, we will consider the following two dimensional problem as a basis for the discussion of some of the latent advantages and disadvantages of the FEM, BEM and hybrid approach.
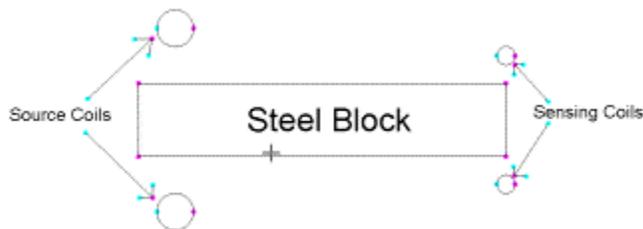

Figure 1: Sample Geometry

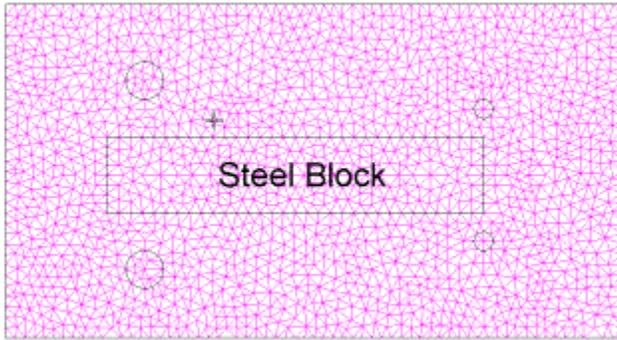Using a pure finite element approach the discretization of the problem would appear as follows:

Figure 2: Finite Element Discretization

Each of the triangles is a finite element and the points where they join are referred to as nodes. If the basis functions α_i are nodal basis functions then the A_i are the values of the magnetic vector potential at each of the nodes. Of course for a vector potential in 3D there will be three values at each node corresponding to each of the Cartesian coordinate directions. In 2D there is simply one unknown at each node point.

Thus to solve a problem with finite elements, the entire domain of the problem has to be discretized. Two other key points should also be noted here. First is that the field does not end where the finite element mesh does. Thus the real problem can never be completely modeled as the domain has to be truncated at some artificial location. The question is how large must the enclosing boundary be in order to get a good approximation to the real problem. As well what should the boundary condition be on that boundary. The simplest method is to set a zero potential boundary condition on the bounding surface. Other methods can sometimes be used to reduce the effect truncating the region but it is a limiting factor for electromagnetic problems.

Another key point is that the solution is the magnetic vector potential. Generally this parameter is of little interest and what is really desired is the magnetic field density or intensity. To calculate the magnetic field density the vector potential must be differentiated. That is

$$\vec{B} = \nabla \times \vec{A}$$

From a numerical perspective differentiation is always an inherently unstable operation to do. Although the solution for $\vec{A}$ may appear to be quite good, it's derivative will always be considerably worse. We shall compare these two key points with the BEM technique later.

BEM METHOD

As with the finite element method we start with the general operator notation

$$L\phi = g$$

For boundary elements

L is an integral operator and ϕ is the solution which is normally a scalar or vector source. The magnetic vector potential is governed by the integral equation

$$\vec{A}(\vec{r}) = \int_S G(\vec{r}, \vec{r}')\vec{K}(\vec{r}')ds' + \int_V G(\vec{r}, \vec{r}')J(\vec{r}')dv'$$

where $G(\vec{r}, \vec{r}')$ is the free space Green's function, $J(\vec{r})$ is the known volume source, and $\vec{K}(\vec{r}')$ is the unknown surface source to be determined. The operator is the integral operating on $\vec{K}(\vec{r}')$.

$\vec{K}(\vec{r}')$

$$G(\vec{r}, \vec{r}') = \ell n \frac{k}{|\vec{r} - \vec{r}'|}$$

for 2D and

$$G(\vec{r}, \vec{r}') = \frac{1}{|\vec{r} - \vec{r}'|}$$

in 3D.

$\vec{r}$ is the location where $\vec{A}$ is being calculated and $\vec{r}'$ is the location of the source terms. Rearranging to put it in operator form

$$\int_S G(\vec{r}, \vec{r}') \vec{K}(\vec{r}') ds' = \vec{A}(\vec{r}) - \int_v G(\vec{r}, \vec{r}') \vec{J}(\vec{r}') dv$$

In this case the unknown is $\vec{K}(\vec{r}')$ and the source term is $\vec{A}(\vec{r}) - \int_v G(\vec{r}, \vec{r}') J(\vec{r}') dv$.

The physical meaning of this equation can best be described with the aid of our original two dimensional drawing.
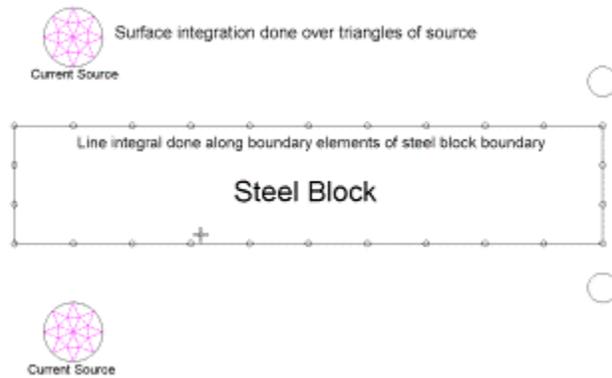

Figure 3: Visual Representation of Integration Method

Part of the contribution to the value of $\vec{A}(\vec{r})$ in two dimensions is done by evaluating the surface integral

$$\int_S G(\vec{r}, \vec{r}') \vec{J}_z(\vec{r}') ds'$$

where $\vec{J}_z(\vec{r}')$ is known and is thus part of the source term. The remaining contribution is calculated by evaluating the line integral

$$\oint_C G(\vec{r}, \vec{r}') \vec{K}_z(\vec{r}') dl'$$

The analogy in three dimensions would be a volume and surface integral. The actual formulation to solve for $\vec{K}(\vec{r}')$ is not included here, but similar to the finite element method a Galerkin approach can be used to define an inner product. In the boundary element method the inner product enforces the continuity of $\vec{H}$ across a boundary. This is simply $\hat{n} \times (\vec{H}_2 - \vec{H}_1) = \vec{K}$. The details of the boundary element formulation have obviously been omitted and can be found in many sources. As with the very brief discussion on finite elements a couple of key points should be noted about boundary elements.

First of all the entire domain does not need to be discretized but only the boundaries of the magnetic materials. This is true when the materials can be assumed linear. For nonlinear materials additional unknowns may be required within the volume. This will be discussed later.

The second key factor is that the region does not need to be artificially truncated. Theoretically the field can be calculated as far away from the problem as desired (including infinity).

The third feature is that once $\vec{K}(\vec{r}')$ is known, the magnetic potential and the magnetic density are calculated by integrating the unknown. The numerical process of integration is very stable resulting in complete continuity of the fields. The magnetic field density is calculated by

$$\vec{B}(\vec{r}) = \int_s \nabla \times G(\vec{r}, \vec{r}') \vec{K}(\vec{r}') ds' + \int_v \nabla \times G(\vec{r}, \vec{r}') \vec{J}(r') dv'$$

Thus the field is calculated by integrating the unknown with the curl of the Green's function.

At this point the major drawback of the boundary element method should be discussed. Dealing with nonlinear materials is straightforward using a finite element approach. However, for boundary elements, the main advantage of only having to discretize the boundaries is lost when material is pushed highly into saturation. In this case additional unknowns are required in the volume of the nonlinear material. So for our sample problem the discretization would now appear as follows with additional unknowns required within the nonlinear volume.

Figure 4: Additional Unknowns for Nonlinear BEM

As will be shown later these additional volume calculations are expensive from a calculation point of view.

HYBRID METHOD

The hybrid approach combines the boundary element and finite element methods to solve a specific problem. The idea is, of course, to take advantage of the strengths of each method for a specific problem.

The approach is straightforward but the implementation is quite difficult. First we have to decide which method to use in each volume (region in 2D); either finite element or boundary element. The two methods are then tied together by enforcing the continuity conditions on either $\vec{B}$ or $\vec{H}$ at the boundaries. In general the strategy is to use the boundary elements in all linear regions and finite elements in all nonlinear regions. In some instances, however, it is desirable to use finite elements in linear regions as well. The rule of thumb is if the surface area of a volume is large relative to the volume it encloses, and moderate solution accuracy is acceptable, the finite elements may be the best choice for these regions. Thus for our original model the steel would contain finite elements and the remainder of the problem would be solved using boundary elements. Remember that the subdivisions or 2D elements in the coils are not finite elements but are simply there to do the surface

integration.
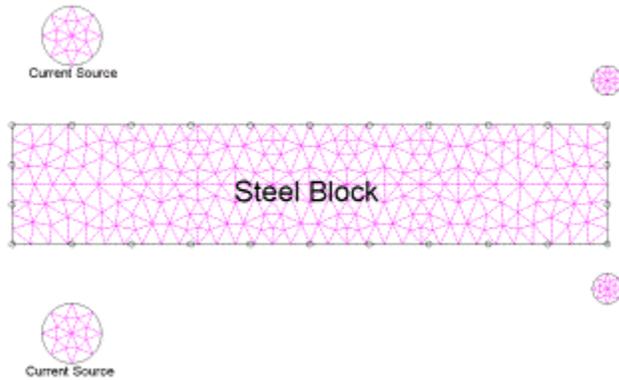
$$\int_S G(\vec{r},\vec{r}')\vec{J}_z(\vec{r}')ds'$$

Current Source

Steel Block

Figure 5: First Hybrid Model

Current Source

If for some reason we chose to use finite elements for the whole problem except the region outside some bounding box where we would use boundary elements then the discretization would appear as

Current Source
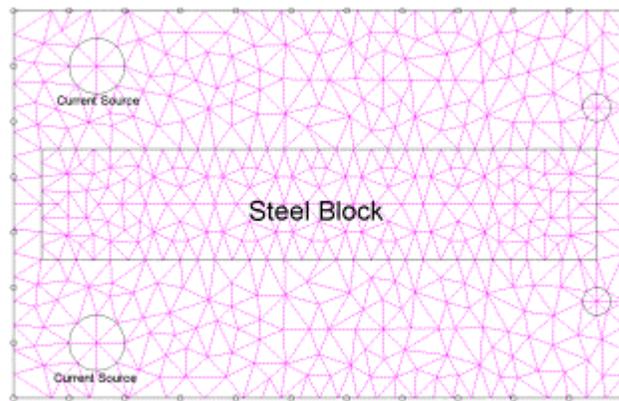
Steel Block

Current Source

Figure 6: Second Hybrid Model

The clear advantage to both of these models is that the exterior region is not artificially truncated. If very high accuracy was required in all the exterior space then the first hybrid model would be chosen. If speed was of primary concern then the second hybrid model would be preferable.

SUMMARY OF THE TWO METHODS

In general both methods have some major strengths and some major drawbacks. These can be easily summarized.

For boundary elements:

1. Extremely high accuracy is attainable for fields. This is due to the field being calculated by integrating the solution.
2. The problem does not have to be artificially truncated and a boundary condition applied to the artificial boundary.

3. For linear problems unknowns are only located on the boundaries of the problem. This radically reduces mesh generation time and storage requirements.
4. The biggest disadvantage of boundary elements is the inability to handle nonlinear problems efficiently. For weakly nonlinear problems the method is satisfactory but for highly nonlinear problems the solution time is excessive.
5. For some classes of problems the solution time for post-processing calculations can be longer than those for finite elements.

For finite elements:

1. The method is easily applied to all types of problems unlike the boundary element method where a different kernel or Green's function may be required for each domain of the problem.
2. Nonlinear problems of all types are readily handled. The nonlinear regions have to be discretized for both differential and integral formulations. Thus the advantage of only discretizing the boundaries of the problem for boundary elements is lost. Very efficient nonlinear methods such as Newton-Rhapson are easily employed within the finite element system of equations.
3. Provided a good mesh can be created the finite element implementation is quite straightforward. Inherently integral equations are very difficult to implement for general curved surfaces due to the singular kernel.
4. The field is trivial to calculate once the problem has been solved. As the solution is known at the nodes of the finite element mesh, it is easily interpolated to any point within the finite element mesh.
5. For problems where the surface area is large compared to the volume it is contained in the solution time can be very good compared to boundary element formulations.
6. The converse to (5) is also true. For problems where a huge amount of volume must be discretized relative to the total surface area the finite element method can be extremely inefficient if it can be applied at all.
7. Differentiating the solution to get the field is a numerically poor process. So, for example, to calculate we have to take the curl . The plots of will appear quite smooth but will have discontinuities which are artifacts of the method due to differentiation.
8. The amount of data required to be stored is large compared to that needed for boundary elements.
9. The "action at a distance" concept is only applicable to equivalent source methods. Thus, for example, forces and torques cannot be calculated by Amperian currents using finite elements.

Three sample problems are shown below: one suited ideally for each of the methods (BEM, FEM, and Hybrid).

The first problem is a magnet testing system consisting of a magnet and measure coils. This problem is ideally suited for boundary elements. Magnet field density values of less than 10 parts per million can be attained with extremely few elements (approximately 20). Thousands of finite elements would be needed to solve this problem with relative course field values.
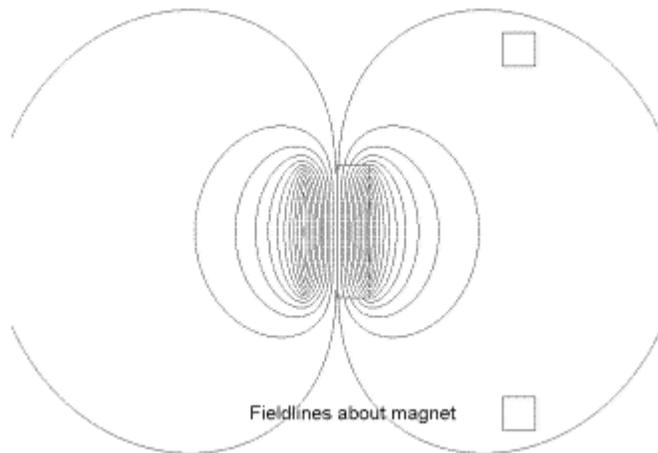
Fieldlines about magnet

Figure 7: Ideal Problem for Pure BEM

The second problem is a highly saturated induction motor. This problem is ideally suited for finite elements as the solution in the exterior space is irrelevant and many parts of the problem are highly saturated. The volume area of the linear region is very small relative to the rest of the problem. This would be the worst case scenario for boundary elements. As well since the torque computation involves the interaction of two sources, accurate torque computations can be arrived at without very precise field computations. This may not hold for certain cogging torque applications.
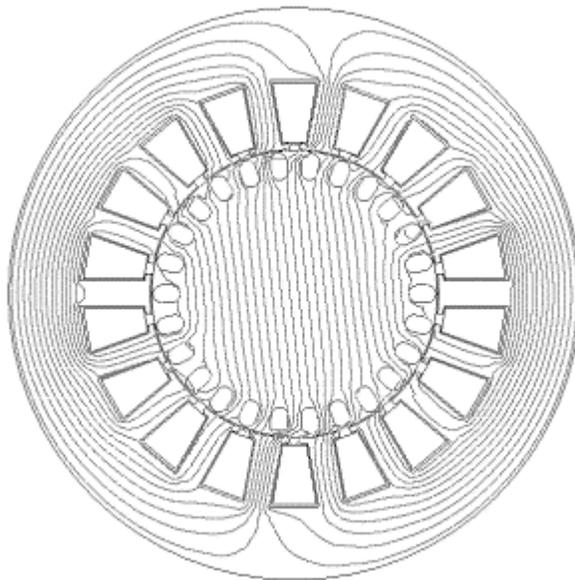

Figure 8: Ideal Problem for pure FEM

The third problem is a magnetic structure for focusing a magnetic field. The problem involves a highly saturated field in the iron with highly accurate field computations required in the air regions. This problem is ideally suited for a hybrid solution where boundary elements are used to calculate the field in the air space and finite elements are used in the nonlinear iron.
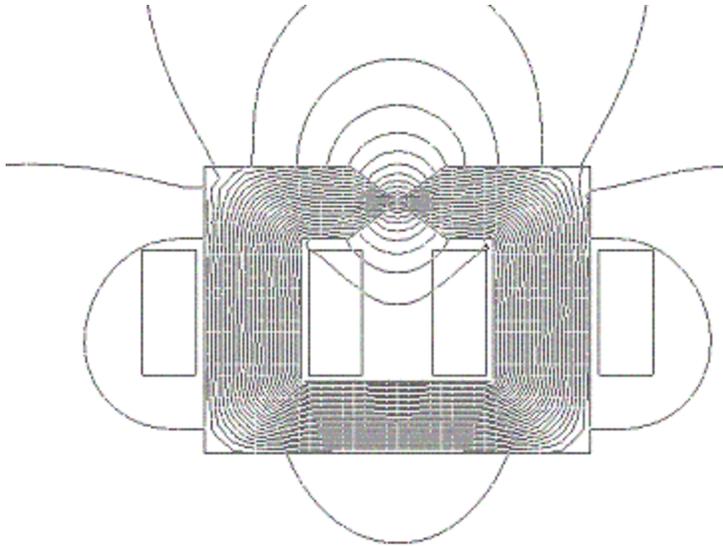
Figure 9: Hybrid Solution
Required

Conclusion

Many publications are widely available on the solution of magnetostatic field problems using finite elements. The method has been widely accepted among many electro-magnetic designers. In addition the boundary element method has been used successfully instead of the traditional finite element approach. The purpose of this paper is to illustrate that the best method to use depends on the problem to be solved. In general the best strategy is to use a hybrid solver in which the advantages of each method can be applied to each region for the entire problem.

As with all numerical techniques, however, there are always subtle difficulties with each method. Although basic guidelines can be established on which method to use it is experience that determines which method will give the desired accuracy in the least amount of time.

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