

# COMPUTER-AIDED MAGNETIC FIELD ANALYSIS USING THE BOUNDARY ELEMENT METHOD

## ABSTRACT

Efficient design of electromagnetic devices requires the analysis of electric and magnetic field distributions. To this end, the boundary element method (BEM) has been shown to be a powerful technique. This paper presents the use of BEM in computer-aided magnetic field analysis. The advantages of BEM over the finite element method (FEM) are stated. The recent introduction of BEM in a generalized, multi-purpose computer-aided design package is discussed. Emphasis is placed on the ease of use and calculation of design parameters. Modeling and solution of problems which include nonlinear materials and permanent magnets are given.

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## **COMPUTER-AIDED MAGNETIC FIELD ANALYSIS USING THE BOUNDARY ELEMENT METHOD**

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Efficient design of electromagnetic devices requires the analysis of electric and magnetic field distributions. To this end, the boundary element method (BEM) has been shown to be a powerful technique. This paper presents the use of BEM in computer-aided magnetic field analysis. The advantages of BEM over the finite element method (FEM) are stated. The recent introduction of BEM in a generalized, multi-purpose computer-aided design package is discussed. Emphasis is placed on the ease of use and calculation of design parameters. Modeling and solution of problems which include nonlinear materials and permanent magnets are given.

### **INTRODUCTION**

Computer-aided analysis of field distribution for evaluating electromagnetic device or component performance has become the most effective way of design. Analytical methods have limited uses and experimental methods are expensive and time consuming.

A typical magnetic field problem is described by defining the geometry, material properties, currents, boundary conditions and the governing field equations. Simulation of fields on the computer require the input of the above, the numerical solution of the field equation and output of desired parameters. The process is repeated until optimum values for the design parameters are obtained.

Numerical solution of the field equations has been an area of intense research. Finite difference method [1], finite element method [2] and boundary element method are used for the numerical solution of the field equations. These methods have been reviewed in an earlier paper [3] and boundary element method has been found to be superior to the rest.

The method of finite differences is the earliest. The disadvantages of this method are among others, the crude modeling of the problem geometry and the large number of unknowns especially in open field problems.

The finite element method provides subsectional polynomial approximations to model the region boundaries. However, in many practical problems, the large amount of input data and unknowns required, the derivative discontinuities in the finite element mesh hence the geometrical model, calculation of potential by interpolation and its derivatives through differentiation and the inability to model infinitely extending regions exactly constitute the major shortcomings of this method. Different techniques have been developed to cater to open region problems. In all of them, the interior of an artificial boundary needs to be packed with finite elements. Large FEM problems still require mini or mainframe computers.

### **BOUNDARY ELEMENT METHOD**

The boundary element method employs an integral equation formulation [4]. Since the unknowns are placed only on the boundary the dimensionality of the problem is reduced by one.

The boundary element method not only produces precise results with far less data as compared to the methods of finite differences and finite elements but also caters to open region problems without any artificial truncation of the region and models problem geometries accurately [5], [6].

An equivalent source, which would sustain the field, is found by forcing it to satisfy prescribed conditions under a free space Green's function which relates the location and effect of the source to any point on the boundary.

The use of Green's function, effectively eliminates the need for a finite element mesh or a finite difference grid.

Once the source is determined, potential and field are computed by integrating the source without interpolation. This provides inherent stability.

In non-linear problems, the BEM still solves for the source of the field and not for its potential. All advantages of calculating the source applies. Only regions with non-linearities contain volume unknowns [7].

### MAGNETIC FIELD EQUATIONS

Electromagnetic field is described by Maxwell's equations. Depending on the materials, boundary conditions and sources, the field can be shown to be governed by one or more partial differential equations. The geometry of the problem usually determines if a two- or three-dimensional analysis will be suitable.

In this paper, we will restrict our attention to two-dimensional stationary or slowly varying magnetic fields.

From Maxwell's equations we have,

$$\nabla \cdot \underline{\mathbf{B}} = 0 \quad (1)$$

and

$$\nabla \times \underline{\mathbf{H}} = \underline{\mathbf{J}} \quad (2)$$

(1) permits us to determine a magnetic vector potential  $\underline{\mathbf{A}}$  as,

$$\underline{\mathbf{B}} = \nabla \times \underline{\mathbf{A}} \quad (3)$$

The constitutive relation in an isotropic material is

$$\underline{\mathbf{B}} = \mu(\mathbf{B}) \underline{\mathbf{H}} \quad (4)$$

Taking the curl of (4) and expanding the right hand side utilizing a well-known vector identity results in,

$$\nabla \times \underline{\mathbf{B}} = \nabla \mu(\mathbf{B}) \times \underline{\mathbf{H}} + \mu(\mathbf{B}) \nabla \times \underline{\mathbf{H}} \quad (5)$$

substituting (3) and (2) into (5) yields,

$$\nabla \times \nabla \times \underline{\mathbf{A}} = \nabla \mu(\mathbf{B}) \times \underline{\mathbf{H}} + \mu(\mathbf{B}) \underline{\mathbf{J}} \quad (6)$$

If  $\mu$  is independent of  $\mathbf{B}$ , we have

$$\nabla \times \nabla \times \underline{\mathbf{A}} = \mu \underline{\mathbf{J}} \quad (7)$$

For a z-directed current density, in two-dimensions, (7) reduces to

$$\nabla^z A_z = \mu J_z \quad (8)$$

which is Poisson's equation.

For problems which contain linear materials only, (8) is the governing field equation. In problems which include non-linear materials, (6) has to be solved iteratively.

Boundary element method entails the conversion of the partial differential equation into an integral equation, as outlined in [3]. The integrand contains the equivalent source and the free space Green's function or its normal derivative depending on the boundary condition. Along interfaces the discontinuity in the tangential component of magnetization is utilized to arrive at the integral equation. Impressed fields and contributions from permanent magnets form part of the forcing function.

The integral equation is discretized along boundaries and interfaces using the Galerkin method. Galerkin method is one of the projection methods which are also called method of weighted residuals or moment methods [5], [6], [8].

Boundaries and interfaces are divided into small sections which are referred to as boundary elements. This discretization results in a set of linear simultaneous equations for the unknown equivalent current density coefficients. The solution of this system of equations, assuming the matrix is not singular, yields the equivalent current density distribution.

The accuracy of the approximation will obviously depend upon the choice of the expansion and testing functions, and the number of them used. These coordinate functions must be linearly independent as linear dependence will result in a singular S matrix.

### COMPUTER-AIDED DESIGN

In a typical computer-aided design cycle, the engineer enters the description of the problem and calculates the parameters of interest. If the values are found unsatisfactory, the design is modified and parameters are recalculated. This cycle is repeated until the desired values are obtained. The efficiency of the procedure is measured by the amount of time required to complete the design.

The factors that affect the efficiency are the ease of use, the accuracy of results, the capabilities and speed of the program.

Ease of use of a package is not a concept totally isolated from the numerical technique used. There are a number of

important factors which are brought about by the solution technique used in the package and which contribute to the ease of use.

Industrial users agree that perhaps the most significant issue is the time required to define the problem. In a boundary element package the user does not mesh every region. This is by far the biggest time saver. FEM requires the engineer to model a problem using a mesh while the BEM model is identical to the blue print.

Again, in a boundary element package, the user does not solve the problem twice in order to calculate forces. Global and local forces are calculated simply by integrating the cross product of the current density with the magnetic induction. Torque calculations are performed with the same ease and accuracy.

### MICROCOMPUTER IMPLEMENTATION

With the advent of powerful microcomputers, computations that were once only possible on mini and mainframe computers, are now possible on microcomputers. In addition, microcomputers offer highly interactive graphics capabilities which can be an invaluable aid in the design of a system.

The geometry, material properties, and other information can be entered with the help of a mouse or digitizer and immediately displayed. This approach minimizes human error and the time required to enter or modify the particular problem.

Early research placed emphasis on the solution time of various problems, when in fact the data preparation can be by far the most time consuming. Data that takes hours to enter can be entered in minutes with the help of a powerful user interface.

The boundary element method has been implemented, on a microcomputer, in the program MAGNETO. The geometry of the problem that can be solved is arbitrary. The conductors may be of finite area or infinitesimally thin. Non-linear materials and permanent magnets are readily catered to.

The solver steps over each element and applies the appropriate inner product. All the integrals are calculated over the simplex  $[0,1]$ .

One difficulty is the integration of the Green's function singularity which occurs when the observation and source points coincide. This problem is easily catered to by dividing out the singularity and using a quadrature scheme containing the form of the singularity. This technique enables very accurate integrations of the singular integrand.

Representing the potential at each point by a phasor, steady state sinusoidal fields can be calculated with ease. For example, multiphase transmission line fields can be calculated by solving a set of real and imaginary equivalent sources for given complex boundary conditions.

The special features of the microcomputer environment: e.g. fast color graphics, color printer, mouse or keyboard entry, math co-processor, hard disk, RAM disk: are fully utilized to create an integrated package which includes problem definition, analysis, data storage and transfer, drafting and presentation capabilities.

The user interface has been designed to require minimal keyboard entry and hand motion. Menus are structured to follow the natural pattern of defining and solving a problem and to incorporate the same sets of commands that operate on different objects. On-line help is provided in every menu.

### APPLICATIONS

Three sample problems which have been solved using MAGNETO are presented below. All calculations were done and all illustrations were generated using MAGNETO on IBM PC type microcomputers. Emphasis is placed on the time required to input the data as well as to solve the problem.

#### A Plunger Problem

The cross-section of the rotationally symmetric plunger configuration is shown in Fig. 1.

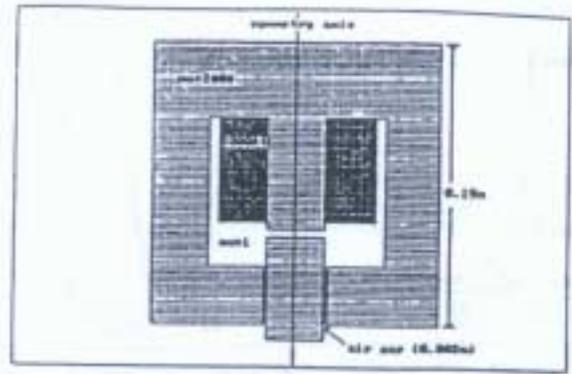


Fig. 1: Cross-section of rotationally symmetric plunger geometry.

Geometry of the problem is entered interactively, using a mouse, in a matter of minutes. Commonly used materials are kept in a table for easy access. MAGNETO, in real time determines simply or multiply-connected regions from

geometry. The materials and currents are assigned by simply selecting the regions. Boundary elements are generated only on segments between dissimilar media.

After the program determines the equivalent current distribution various parameter can be calculated. Fig. 2 shows a contour map of fieldlines.

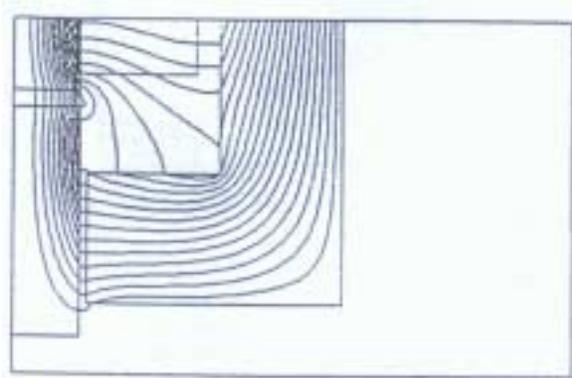


Fig. 2: Contour map of equi-fieldlines for the plunger problem.

The goal of this design is to increase the force acting on the armature. Force is calculated by integrating the cross product of the current density with magnetic induction. In this case, since the equivalent current density is in the angular direction, it is the radial component of magnetic induction which creates the force in z-direction. Fig. 3 shows the graph of radial component of magnetic induction in the air gap.

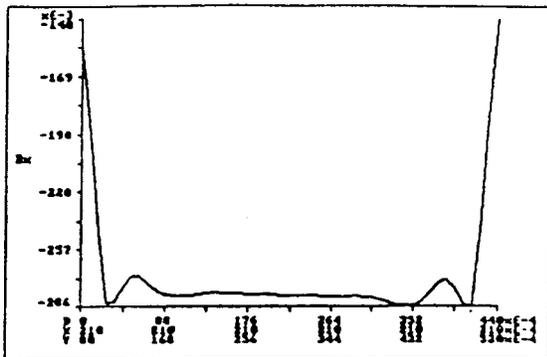


Fig. 3: Graph of radial component of magnetic induction in air gap.

A useful and informative form of display is the three dimensional surface representation of scalar field quantities. Fig. 4 shows the magnitude of magnetic induction in the immediate vicinity of the air gap. This form of display helps visualize the field distribution over an area at a glance.

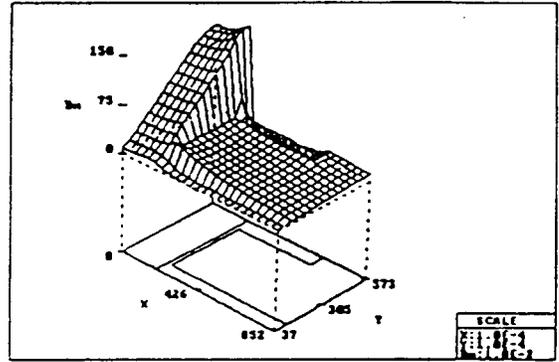


Fig. 4: Surface representation of magnitude of magnetic induction.

### A Commutated DC Machine

The magnetic vector potential and fields are calculated for a commutated DC machine under no load condition, using MAGNETO. This problem, shown in Fig. 5, is taken from [9]

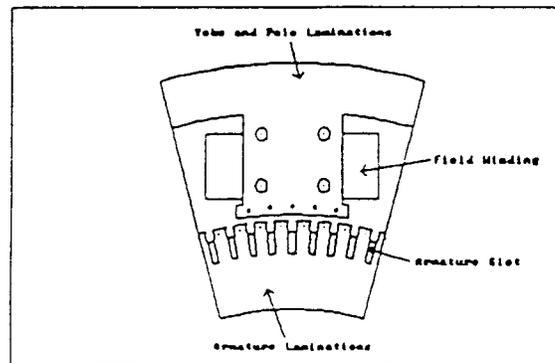


Fig. 5: Pole pitch cross-section of DC machine.

The geometry of the problem consists of arcs and lines. Approximately one hour was required to enter the problem. We believe that, due to the holes of various sizes in the yoke and pole laminations, a large number of finite elements and a much longer input time would be required to model the same problem using a finite element package.

The relative permeability of the yoke, pole and armature laminations is assumed to be 1000. A total of 295 boundary elements were used to solve the problem. The solution time on an IBM-AT was one hour and fifteen minutes. Fig. 6 shows the flux map near the mid gap.

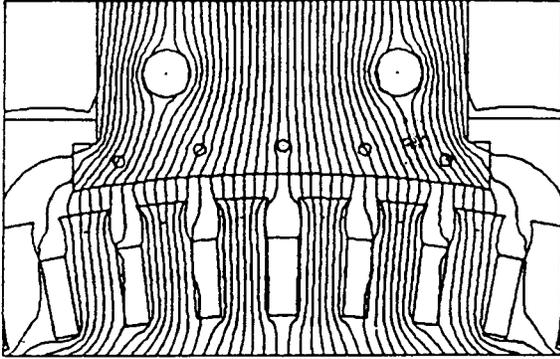


Fig. 6: Flux map near the mid gap for DC machine problem.

The graph of the magnitude of the magnetic induction along the mid gap is shown in Fig. 7. Unlike the results of the finite element analysis, boundary element results are smooth and symmetric about the center line.

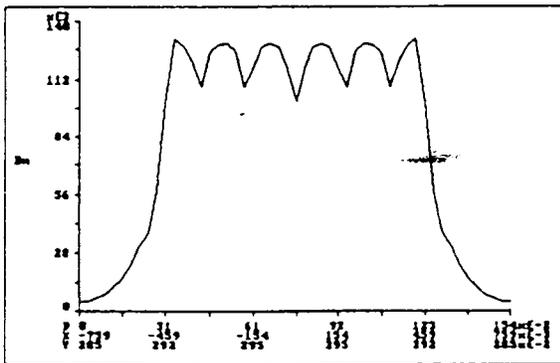


Fig. 7: Graph of magnitude of magnetic induction in mid gap.

#### A Permanent Magnet Motor

The magnetic field distribution and torque are calculated for a permanent magnet motor. The geometry of the motor cross-section and materials are shown in Fig. 8. The coercive force of the magnets is 8320 Oersteds and the remanent field is 8600 Gauss. Using MAGNETO, entering the problem from the blue print took about two hours.

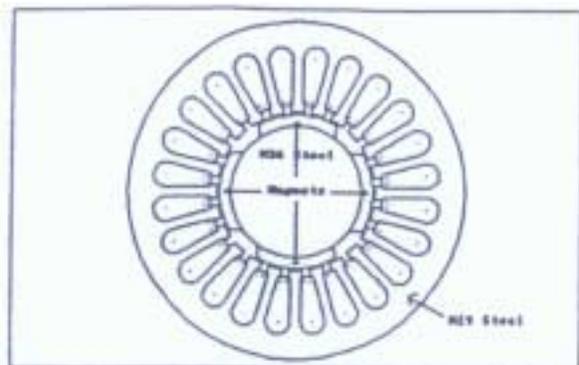


Fig. 8: Cross-section of the permanent magnet motor.

Half plane symmetry of the rotor and stator geometry was utilized for the solution. A total of 253 boundary elements were used. The solution time, including the non-linear iterations, was about 1.5 hours using an IBM PS/2 Model 80. Fig. 9 shows the flux map near the upper part of the motor. The boundary element method calculates both the near and far fields at one time with the same accuracy. One can zoom into any area and obtain accurate results. In the FEM, if one were to zoom into a single element, one would only obtain interpolations from the nodal values of that element.

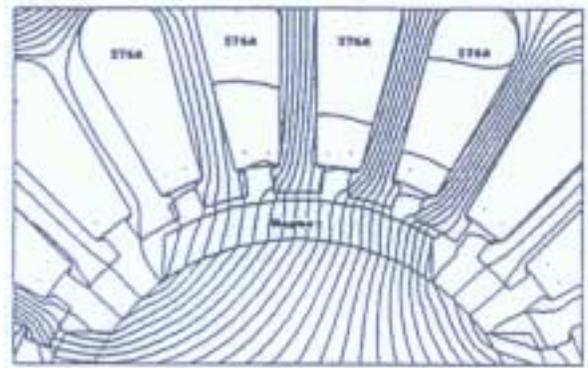


Fig. 9: Flux map near the upper part of the motor.

The calculated torque was 898 lb x in/m and the measured was 866 lb x in/m. The difference which is less than 4% may be attributed to the end effects.

#### CONCLUSIONS

The boundary element method has been shown to be an efficient technique also for the solution of Poisson's equation for non-linear media. This is mainly due to the reduction of one in dimensionality as all the unknowns are located only on the boundaries and interfaces. This differs from the finite difference and finite element methods in which the whole domain must be discretized. The unknown, computed using the boundary element method, is the equivalent source that sustains the field. Once the equivalent source is known any parameter can be derived.

The solution technique used in a package is responsible not only for the accuracy of results but also for the ease of use of the package.

The boundary element method, combined with a highly interactive user interface, automates the computation and analysis of magnetic field distributions for motor, generator, solenoid, speaker, actuator and other magnetic device design.

Problem geometries, materials, currents and boundary conditions can be conveniently described from the conceptual stage and be analyzed to obtain the desired design parameter. Accurate and reliable results are due to the boundary element method and the efficient user interface.

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