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ABSTRACT

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Reprinted from PROCEEDINGS OF THE INTERNATIONAL COIL WINDING CONFERENCE, Rosemont, Illinois, October 5-8, 1987

COMPUTER-AIDED FIELD ANALYSIS OF HIGH VOLTAGE APPARATUS USING THE BOUNDARY ELEMENT METHOD

Y. Bulent Yildir

Integrated Engineering Software Inc.
9-1329 Niakwa Road, Winnipeg, Manitoba, Canada, R2J 3T4

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INTRODUCTION

Calculation of design parameters for high voltage power apparatus usually requires the solution of the electromagnetic field equations in a region B under prescribed boundary conditions on ∂B. The analytical methods are rather restricted to problems with simple geometries. The numerical methods, on the other hand, offering general applicability have been the object of intense research. Especially within the last forty years or so, various numerical methods have been developed owing to the advent of powerful computers.

The common concept in the numerical methods is the reduction of the governing field equation or an equivalent integral formulation into a linear system of equations. These methods can be classified in two categories: the methods where approximations are to be made throughout the region B; and the methods where approximations are to be made only on the boundary ∂B. The finite difference and finite element methods belong to the first category while the boundary element methods belong to the second.

FINITE DIFFERENCE METHOD

The method of finite differences is among the first numerical techniques that offered a broad range of applicability [1]. In this method, utilizing a truncated Taylor series expansion in each coordinate direction, the differential operator is discretized and applied at each point of a rectilinear grid placed on B. The resulting system of equations is solved either iteratively or directly. The disadvantages of the method are, among others, the crude modelling of the problem geometry and the large number of unknowns especially in open field problems.

FINITE ELEMENT METHOD

The method of finite elements uses a variational formulation [2],[3]. In this method, the region B is divided into a finite number of non-separated, non-overlapping, very small sub-areas of some prescribed shape which are termed elements. Element geometries and unknowns are expressed by polynomials with nodal values as coefficients. Relating these approximations to the operator equation through minimizing a functional yields the solution at the nodes. This method provides better means, i.e. subsectional polynomial approximations, to model the region boundaries.

However, in many practical applications, the large amount of input data and unknowns required, the derivative discontinuities in the finite element mesh hence the geometrical model, calculation of potential by interpolation and its derivatives through differentiation, localized large errors, no simple means of checking the accuracy of the solution and the inability to model infinitely extending regions exactly constitute the major shortcomings of the finite element method.

Calculation of capacitance and inductance is also difficult and unreliable with the FEM as these calculations require derivatives of the potential.

The derivative discontinuities in the finite element mesh may be eliminated by using Hermitian or cubic spline elements [4]. Different techniques; e.g. infinite elements, picture frame method [5]; have been developed to cater to open region problems. In all of them, the interior of the artificial boundary needs to be packed with finite elements. Large FEM problems still require mini or mainframe computers.

BOUNDARY ELEMENT METHOD

The methods of the second category solve a boundary integral equation formulation of the problem for some unknowns on ∂B [6]. These methods have been attracting considerable attention for the last twenty years or so because they not only produce precise results with far less data as compared to the methods of finite differences and finite elements but also cater to open region problems without any artificial truncation of the region and model problem geometries accurately. Since the approximations are done only on the boundary , the dimensionality of the problem is reduced by one. Furthermore, usually being bounded and often completely continuous, integral operators as compared to differential operators admit a wider selection of trial functions [7].

Direct methods in this category solve an integral equation formulation for the unknowns directly [8], while indirect methods solve for the source of the unknown [9]. The boundary element method presented in this paper is an indirect method. An equivalent source, which would sustain the field, is found by forcing it to satisfy prescribed conditions under a free space Green's function which relates the location and effect of the source to any point on the boundary.

The use of Green's function, effectively eliminates the need for a finite element mesh or a finite difference grid.

Once the source is determined, potential and field are computed by integrating the source without interpolation. This provides inherent stability. Capacitance, inductance, and other parameters can be calculated by integrating the free charge, which is derived from the equivalent source [10]. Provided the problem is piecewise homogeneous, the equivalent source is located only on the boundaries and interfaces of different media.

In non-linear problems, the BEM still solves for the source of the field and not for its potential. All advantages of calculating the source applies. Only regions with non-linearities contain volume unknowns [11].

PHYSICAL BASIS

In this paper we restrict our attention to problems where a static approximation in two dimensions is adequate.

In an electrostatic field,

$$\nabla \times \underline{E} = 0 \qquad (1)$$

so that $\underline{E}$ is irrotational and hence conservative which is a necessary and sufficient condition for the existence of a potential ϕ in the form

$$\underline{E} = -\nabla\phi \qquad (2)$$

Again from Maxwell's equations, in a source-free region

$$\nabla \cdot \underline{D} = 0 \qquad (3)$$

The constitutive relation for a linear, isotropic region of dielectric constant ϵ is

$$\underline{D} = \epsilon\underline{E} \qquad (4)$$

If the region is homogeneous, combining (2), (3) and (4)

$$\nabla^2\phi = 0 \qquad (5)$$

which is Laplace's equation.

INTEGRAL EQUATION FORMULATION

In a bounded region B with a piecewise smooth boundary ∂B, application of Green's theorem [12]

$$\iint_B (\phi \nabla^2 G - G \nabla^2\phi)\, dB = \int_{\partial B} (\phi \frac{\partial G}{\partial n} - G \frac{\partial \phi}{\partial n})\, dr' \qquad (6)$$

to the unknown potential ϕ and the free-space Green's function [13]

$$G(\underline{r},\underline{r}') = \ln \frac{k}{|\underline{r}-\underline{r}'|} \qquad (7)$$

satisfying

$$-\nabla^2 G = 2\pi\delta(\underline{r}-\underline{r}') \qquad (8)$$

where δ is the Dirac delta function, yields

$$\int_{\partial B} (\phi \frac{\partial G}{\partial n} - G \frac{\partial \phi}{\partial n})\, dr' = \begin{cases} -2\pi\phi(\underline{r}) & \underline{r} \in B \\ 0 & \underline{r} \in B_o \end{cases} \qquad (9)$$

k is a constant chosen such that $k > \max|\underline{r}-\underline{r}'|$ which ensures that the Green's function is strictly positive throughout B.

B_o is the region exterior to B. The validity of (9) can be extended to an infinite region provided that ϕ and G are regular at infinity [12]. Thus for the exterior region

$$-\int_{\partial B} (\phi_o \frac{\partial G}{\partial n} - G \frac{\partial \phi_o}{\partial n})\, dr' = \begin{cases} 0 & \underline{r} \in B \\ -2\pi\phi(\underline{r}) & \underline{r} \in B_o \end{cases} \qquad (10)$$

Adding equations (9) and (10) results in

$$\int_{\partial B} (\phi-\phi_o) \frac{\partial G}{\partial n} - G\, (\frac{\partial \phi}{\partial n} - \frac{\partial \phi_o}{\partial n})\, dr' = \phi(\underline{r}) \qquad (11)$$

The choice of $\phi = \phi_o$ and

$$\sigma(r') = (\frac{\partial \phi}{\partial n} - \frac{\partial \phi_o}{\partial n}) \qquad (12)$$

gives

$$\int_{\partial B} G(\underline{r},\underline{r}')\, \sigma(\underline{r}')\, dr' = \phi(\underline{r}) \qquad (13)$$

which is the simple layer integral equation formulation for the Laplace's equation [6],[9].

The integrand contains the distributed source and the free space Green's function. From (13), given the source configuration, the potential can be found everywhere in the region. Usually, however, the source is not known but the potential or its normal derivative are specified on the boundary and we seek an equivalent source that will sustain these conditions. Once the equivalent source is known, any field value or parameter can be calculated.

For Dirichlet boundaries the equation to be enforced is (13). For exterior Dirichlet problems to construct an acceptable solution the boundary decomposition [6]

$$\phi(\underline{r}) = \int_{\partial B} G(\underline{r},\underline{r}')\sigma(\underline{r}')d\underline{r}' + C \qquad \underline{r} \in \partial B \qquad (14)$$

is introduced where C is a constant to be determined under the side condition,

$$\int_{\partial B} \sigma(\underline{r}')d\underline{r}' = 0 \qquad (15)$$

(15) is necessary for the logarithmic potential to be regular at infinity.

For Neumann boundaries a Fredholm equation of the second kind results

$$\phi'(\underline{r}) = \int_{\partial B} G'(\underline{r},\underline{r}')\sigma(\underline{r}')dr' + m\sigma(\underline{r}) \qquad \underline{r} \in \partial B \qquad (16)$$

where $\phi'(\underline{r})$ and $G'(\underline{r},\underline{r}')$ are the normal derivative of potential and the Green's function with respect to the unprimed variable.

Along any interface the continuity of flux density is enforced yielding

$$(\epsilon_1 - \epsilon_2)\int_{\partial B} G'(\underline{r},\underline{r}')\sigma(\underline{r}')dr' + (\epsilon_1 + \epsilon_2)\sigma(\underline{r}) = 0 \qquad (17)$$

where ϵ_1 and ϵ_2 are the permittivity values of the materials forming the interface.

To solve the above integral equations for the equivalent source the Galerkin method is used.

PROJECTION METHODS

Projection methods are also called method of weighted residuals or moment methods [8],[9],[14]. Consider the operator equation

$$L\sigma = g \qquad (18)$$

where L is assumed to be a linear operator which maps σ to g uniquely. Normally L and g are known and we have the deterministic problem of finding σ. That is, we are required to solve

$$\sigma = L^{-1}g \qquad (19)$$

where L^{-1} is assumed to exist and that the solution for σ is unique.

Let the solution be expanded by the series of functions in the domain of the operator and let $a_1, a_2, a_3, \ldots$ be coefficients such that

$$\sigma(x) = \sum_{n=1}^{m} a_n \beta_n(x) \qquad (20)$$

For an exact solution the expansion functions must form a complete set which is usually infinite in number. Rewriting (18) as

$$L\sigma(x) - g(x) = 0 \qquad (21)$$

and substituting the expansion functions to approximate the potential, the residual is

$$R = \sum_{n=1}^{m} a_n L\beta_n(x) - g(x) \qquad (22)$$

which is equal to zero only if the coefficients and expansion functions can be found such that they are the exact solution. In the projection method the coefficients are found in such a way that the residual is forced to be zero - giving the best approximation.

A suitable inner product is taken with the residual and some prescribed functions over the range of the operator. These functions are called weighting functions, or more descriptively, testing functions. The inner product is defined by

$$\langle w_m, R \rangle = \int R\, w_m\, ds \qquad m=1,2,3,\ldots \qquad (23)$$

where $w_1, w_2, w_3, \ldots$ are the testing functions. The inner product is set to zero forcing the residual to be orthogonal to the testing functions

$$\langle w_m, R \rangle = 0 \qquad (24)$$

Substituting (22) into (24) and rearranging yields

$$\sum_{n=1}^{m} a_n \langle w_m, L\beta_n(x) \rangle = \langle w_m, g(x) \rangle \qquad (25)$$

For a solution of (25) we approximate (20) by a finite sum. Eq. (25) is then a finite set of linear equations which can be put in matrix form as

$$S\underline{a} = \underline{b} \qquad (26)$$

where

$$s_{mn} = \langle w_m, L\beta_n \rangle \qquad b_m = \langle w_m, g \rangle \qquad (27)$$

Assuming the matrix is not singular it may be inverted yielding the coefficients. These coefficients may then be substituted into (20) giving an approximate (on rare occasion an exact) solution for the charge.

The accuracy of the approximation will obviously depend upon the choice of the expansion and testing functions, and the number of them used. These coordinate functions must be linearly independent as linear dependence will result in a singular S matrix.

The particular choice of the expansion functions being the same as the testing functions is called Galerkin's method.

Boundaries are discretized into individual sections which are referred to as boundary elements. The expansion and testing functions, as well as the geometry, are specified on an element-by-element basis. Coefficients of the expansion functions are normally defined at nodes on the element. Each node is associated with a particular expansion function. Using linear shape functions,

$$\alpha_1 = 1 - \xi \qquad \alpha_2 = \xi \qquad (28)$$

the charge over each element is expressed as

$$\sigma = \sum_{i=1}^{m} \sigma_i \, \alpha_i(\xi) \tag{29}$$

where m=2 as linear elements are used.

Using Lagrange quadratic shape functions

$$\begin{aligned} \alpha_1 &= 2\xi^2 - 3\xi + 1 \\ \alpha_2 &= 4(\xi - \xi^2) \\ \alpha_3 &= 2\xi^2 - \xi \end{aligned} \tag{30}$$

over the domain [0,1], global positions in cartesian coordinates are specified parametrically over each element as

$$x = \sum_{i=1}^{m} \alpha_i(\xi) x_i \qquad y = \sum_{i=1}^{m} \alpha_i(\xi) y_i \tag{31}$$

We wish to determine

$$\langle w_m, L\sigma_n \rangle = \langle w_m, b \rangle \tag{32}$$

which can be put in vector notation as

$$\langle \underline{\alpha}, L\underline{\alpha}^T \rangle \underline{\sigma} = \langle \underline{\alpha}, g \rangle \tag{33}$$

The operator L is dependent upon the boundary conditions where the inner product is being calculated.

MICROCOMPUTER IMPLEMENTATION

With the advent of powerful microcomputers, computations that were once only possible on mini and mainframe computers, are now possible on microcomputers. In addition, microcomputers offer highly interactive graphics capabilities which can be an invaluable aid in the design of a system.

The geometry, material properties, and other information can be entered with the help of a mouse or digitizer and immediately displayed. This approach minimizes human error and the time required to enter or modify a particular problem.

Early research placed emphasis on the solution time of various problems, when in fact the data preparation can be by far the most time consuming. Data that takes hours to enter by hand can be entered in minutes with the help of a powerful user interface.

The boundary element method presented above has been implemented, on a microcomputer, in the program ELECTRO. The geometry of the problem that can be solved is arbitrary. The conductors may be of finite area or infinitesimally thin.

The solver steps over each element and applies the appropriate inner product. All the integrals are calculated over the simplex [0,1].

One difficulty is the integration of the Green's function singularity which occurs when the observation and source points coincide. This problem is easily catered to by dividing out the singularity and using a quadrature scheme containing the form of the singularity. This technique enables very accurate integrations of the singular integrand.

Representing the potential at each point by a phasor, steady state sinusoidal fields can be calculated with ease. For example, multiphase transmission line fields can be calculated by solving a set of real and imaginary equivalent sources for given complex boundary conditions.

The special features of the microcomputer environment; e.g. fast color graphics, color printer, mouse or keyboard entry, math coprocessor, hard disk, RAM disk; are fully utilized to create an integrated package which includes problem definition, analysis, data storage and transfer, drafting and presentation capabilities.

The user interface has been designed to require minimal keyboard entry and hand motion. Menus are structured to follow the natural pattern of defining and solving a problem and to incorporate the same sets of commands that operate on different objects. On-line help is provided in every menu.

The use of the boundary element method also benefits the user interface. Geometry definition is not built around a mesh and the accuracy of results is easily checked by sound means.

One has very powerful options to test the accuracy of a solution. On boundaries, the calculated and assigned conditions can be compared. Along interfaces, the calculated and actual field discontinuities can be checked. One could test the field values inside conductors. According to maximum principle of harmonic functions, the largest errors occur on boundaries. Hence these checks indicate the largest error in a solution.

In the FEM, since the results are provided by interpolation no such simple and quick ways of checking the accuracy of a solution exists - on boundaries one would obtain exactly what was assigned.

APPLICATIONS

A number of problems which have been solved both by FEM and BEM are presented below. All BEM calculations were done using ELECTRO on IBM PC type microcomputers. Emphasis is placed on the time required to input the data as well as to solve the problem.

A Bus Bar Problem

Maximum value of electric field magnitude (kV/mm) is calculated for a pair of rectangular bus bars shown in Fig. 1.

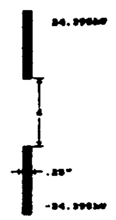

Fig. 1: Rectangular bus bar configuration.

Table 1 presents the results for varying distance d and corner radius r. Dimensions are in inches.

Table 1
Bus bar problem results.

d	r	ELECTRO	FEM
2.0	.125	2.6	2.7
2.0	.0625	2.9	3.3
2.0	.0	9.6	5.9
2.5	.0	8.6	5.0
2.75	.0	8.2	4.5
3.0	.0	7.9	4.3
3.5	.0	7.3	3.8

When r = 0, BEM consistently gives higher values as the number of elements is increased. This is expected as the field is infinite at the corner. When corners are rounded, FEM gives values higher than BEM, possibly due to the artificial truncation of the open region.

A High Voltage Test Arrangement

Electrostatic potentials and fields are calculated for a high voltage test arrangement using ELECTRO. This rotationally symmetric problem is extracted from [15]. We modeled the saw-tooth like meridional profile with splines. The problem geometry and dimensions are shown in Fig. 2.

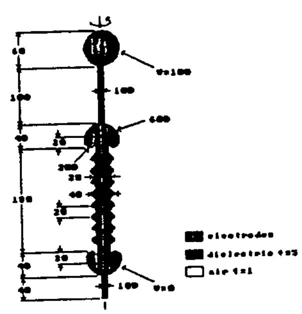

Fig. 2: Two metallic electrodes separated by a dielectric piece.

The finite element mesh contained 1424 nodes (119 on the boundaries) and is made up of 575 elements (154 quadrilaterals and 421 triangles) and the data preparation took 3 days. The c.p.u. time needed for the solution, using an H 66/60 computer, was 6 minutes [15].

Less than fifteen minutes was required to enter all the input data, using ELECTRO. A total of forty boundary elements were used to solve the problem. The solution time was three minutes and twenty seconds using an IBM PC computer. Fig. 3 shows the equipotential contours.

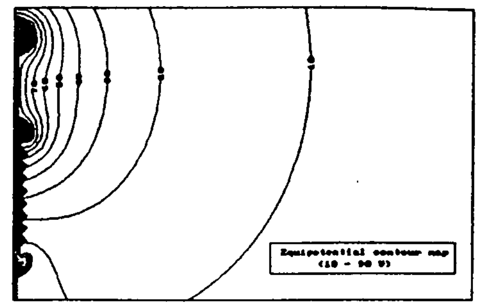

Fig. 3: Equipotential contour map for high voltage test arrangement.

Field Analysis for an Epoxy-SF6 Bushing

Electrostatic fields and potentials are calculated for an epoxy-SF6 bushing. Fig. 4 shows the FEM model which is, using about 1500 finite elements, a very rough approximation of the actual problem.

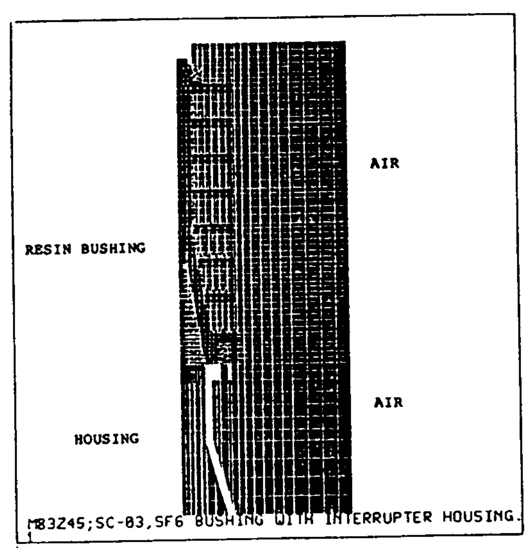

Fig. 4: Finite element model of SF6 resin bushing and housing.

Fig. 5 shows the equipotential contours which are non-physical. Consequently, the electric field values, which are calculated by differentiating the potential, are invalid.

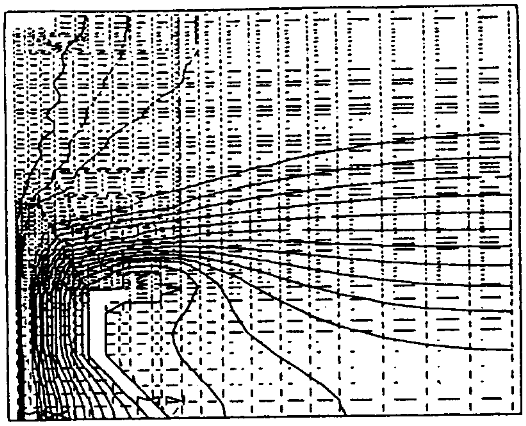

Fig. 5: Equipotential contours for SF6 resin bushing in steps of 5 units, obtained using FEM.

The BEM analysis of the same problem is performed using 200 boundary elements. The problem is modeled with exact data. Fig. 6 shows the dimensions near the region of interest.

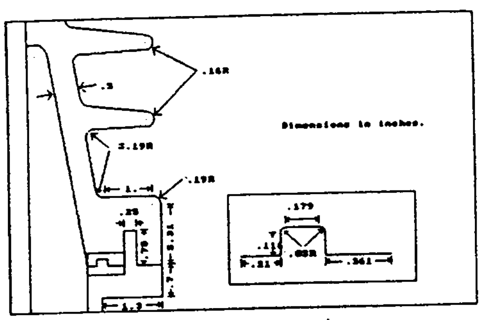

Fig. 6: Dimensions near lower skirt, for SF6 resin bushing problem (BEM model).

Reading the blue print and modeling the problem took two hours. The solution time was 59 min. on an IBM PC/AT. Fig. 7 shows the equipotential contours. The BEM calculates both the near and far fields at one time with the same accuracy. One can zoom into any area and obtain accurate results. In the FEM, if one were to zoom into a single element, one would only obtain interpolations from the nodal values of that element.

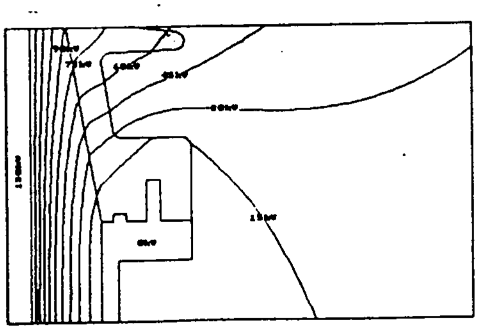

Fig. 7: Equipotential contours near aluminum flange in steps of 15kV, obtained using BEM.

FEM requires the engineer to model a problem using a mesh while the BEM model is identical to the blue print. Electrostatic field magnitude is largest on the surface. Therefore, most relevant information is obtained from graphs of normal and tangential fields along boundaries and interfaces. Fig. 8 shows the electric field magnitude along SF6-epoxy interface.

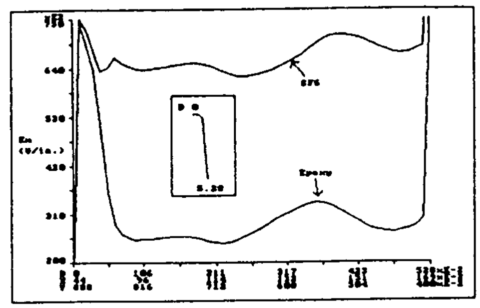

Fig. 8: Magnitude of electric field along SF6-epoxy interface, obtained using BEM.

CONCLUSIONS

The boundary element method has been shown to be an efficient technique for the solution of Laplace's equation for piecewise homogeneous media. This is mainly due to the reduction of one in dimensionality as all the unknowns are located only on the boundaries and interfaces. This differs from the finite difference and finite element methods in which the whole domain must be discretized. The unknown, computed using the boundary element method, is the equivalent charge that sustains the field. Once the equivalent charge is known any parameter can be derived.

The boundary element method, combined with a highly interactive user interface, automates the computation and analysis of field distributions around high voltage power apparatus. Problem geometries, materials and boundary conditions can be conveniently described from the conceptual stage and be analyzed to obtain the desired design parameter. Accurate and reliable results are due to the boundary element method and the efficient user interface.

ACKNOWLEDGEMENTS

I gratefully acknowledge the courtesy of companies whose results and data I have been able to use in this paper. In particular, I am thankful to A.B. Chance Co., Centralia, Missouri for providing me with the results of the bus bar problem.

This research, in part, was supported by an IRAP-M grant from National Research Council of Canada.

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