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## A COMPUTER PROGRAM FOR 2D/RS EDDY CURRENT PROBLEM BASED ON BOUNDARY ELEMENT METHOD

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### ABSTRACT

A computer program, OERSTED, for simulating and analyzing two-dimensional or rotational symmetric eddy current problem with the steady-state sinusoidal time-varying source excitation has been developed at the Integrated Engineering Software (IES). Like other widely used CAE tools developed at IES, OERSTED is a software package based on the most advanced Boundary Element Method (BEM) technique. OERSTED provides a very friendly user interface, exact modelling of boundaries and accurate results. The program has the capability of calculating the skin and proximity effect, resistance and inductance, eddy current, electric and magnetic fields, Joule loss, forces and torques. In this paper, the boundary integral equation formulation and its numerical solution are described. Several application examples are presented to show the accuracy, efficiency and reliability of the program.

### INTRODUCTION

Eddy currents are found in any conducting material which is subjected to a time-varying magnetic field. They occur in all types of electrical equipment, for example, induction motor, power cable, coil, etc. It is often possible to put these currents to good use, such as induction heating, where the ohmic losses are sufficient to melt the metal. On the other hand, for certain applications, steps may have to be taken to reduce the effects of the eddy currents. For example, the cores of transformers are laminated in order to reduce the loss and enable the core to carry the required magnetic flux, which would otherwise be almost completely inhabited by the eddy-current reaction field. Therefore, the analysis of eddy currents becomes very important in the process of electromagnetic equipment design.

Analytical methods for the analysis of eddy currents are limited to only several very simple geometries. Some approximate methods are available, but are often not acceptable. Experimental methods are very expensive and time consuming. On the other hand, numerical methods for simulating and analyzing electromagnetic (EM) fields have become the most effective and powerful tools to help engineers visualize and manipulate EM fields and thereby, to

design products without the extensive testing and redesign formerly needed to accommodate unforeseen EM effects.

The numerical treatment of most linear or nonlinear EM field problems can be effectively achieved through a Boundary Element Method (BEM). As compared with the domain-type methods, such as Finite Element Method (FEM) or Finite Difference Method (FDM), the BEM has two salient advantages: 1) the dimensions of the problem are effectively reduced by one; 2) the analysis is equally applicable to bounded or unbounded regions. Particularly, in the case of analysis of eddy current problems with high conductivity and frequency, the strong skin effect makes domain-type methods very difficult and expensive to use. One must use very fine mesh or grid in the region close to the conductor boundary in order to accurately calculate the field distribution which attenuates exponentially away from the boundary. BEM, in contrast to other methods, is ideal for this application in that only the material interfaces need to be included in the formulation. Then the field distribution can be accurately calculated by using the equivalent source through a Green's function.

Recently, a computer program, OERSTED [1], for simulating and analyzing two-dimensional or rotational symmetric eddy current problems with the steady-state sinusoidal time-varying source excitation has been developed at the Integrated Engineering Software (IES). Like other widely used CAE tools developed at IES, OERSTED is a software package based on the most advanced BEM technique. The package provides a very friendly user interface, exact modelling of material boundaries and accurate results. It has the capability of calculating the skin and proximity effects, induced current, EM fields, Joule loss, impedance, forces and torques. The program can handle both linear and nonlinear materials, and wide range of frequency including the magnetostatic case.

In this paper, a dual simple-layer boundary integral equation and its numerical solution are described. Several application examples are presented to show the accuracy, efficiency and reliability of the program.

### INTEGRAL EQUATION FORMULATION

Consider a two-dimensional conductor of arbitrary cross section embedded in free space. For simplicity, we assume that the conducting material is linear, isotropic and homogeneous. The conductor has a conductivity  $\sigma$  and magnetic permeability  $\mu$ . In the case of TM polarization where current flows in the  $z$  direction, the 2D time-harmonic diffusion equation in terms of  $z$  component magnetic vector potential  $A$  in the conductor may be written as

$$(\nabla^2 + K^2)A = -\mu J^s \quad (1)$$

where  $K^2 = -j\omega\mu\sigma$ , and  $J^s$  is the source current density. For the eddy current problem,  $J^s$  is an unknown constant [2]. In a somewhat unconventional way, we may introduce two additional potentials according to [3]:

$$A^t = A + A^s \quad (2)$$

where  $A^t$  represents the total field, and  $A^s$  represents the source component which is a gradient field. Similarly, the total current density  $J^t$  has two components, eddy current  $J^e$  and source current  $J^s$ ,

$$J^t = J^e + J^s \quad (3)$$

where

$$J^s = -j\omega\sigma A^s \quad (4)$$

Upon substituting (4) into (1) and using the fact that  $A^s$  is a constant in the conductor, we can express the governing equation in the conducting region as

$$(\nabla^2 + K^2)A^t = 0 \quad (5)$$

In free space, the governing equation is

$$\nabla^2 A = 0 \quad (6)$$

From the equivalence principle [4], one can show that the fields in the two regions may be obtained by using two surface current distributions  $J_1$  and  $J_2$  on the surface  $S$  which is the interface between the conductor and the air. From (5), the total magnetic vector potential in the conducting region can be expressed as

$$A_1^t(r) = \mu \int_S G(r, r') J_1(r') dr' \quad (7)$$

where  $r$  and  $r'$  are the field and source points, respectively. And from (6), the reaction potential in free space can be written as

$$A_2(r) = \mu_0 \int_S G_2(r, r') J_2(r') dr' \quad (8)$$

In above equations, subscripts 1 and 2 denote conducting region and air, respectively. The Green's functions in (7) and (8) are, respectively,

$$G_1(r, r') = \frac{1}{4j} H_0^{(2)}(k|r-r'|) \quad (9)$$

and

$$G_2(r, r') = -\frac{1}{2\pi} |n|r-r'| \quad (10)$$

In (9),  $H_0^{(2)}$  is the zero order Hankel function of the second kind.

By enforcing the  $A$  and tangential  $H$  continuity conditions on the boundary  $S$ , one obtains a dual simple-layer boundary integral equation as

$$A_1^t(r; J_1) - A_1^s - A_2(r; J_2) = 0, \quad r \in S \quad (11)$$

$$nx(H_1(r; J_1) - H_2(r; J_2)) = 0, \quad r \in S \quad (12)$$

in which  $n$  is the normal vector of  $S$ , and

$$H_i = \frac{1}{\mu_i} \nabla A_i \times \hat{z}, \quad i=1,2 \quad (13)$$

An additional equation is obtained by enforcing the conservation condition

$$\oint_S H_2 \cdot dl - I \quad (14)$$

where the integration is performed along the conductor surface  $S$ . Combining (14) with (11) and (12), we can obtain the solution for the unknowns  $J_1$ ,  $J_2$ , and  $A_1^s$ . Other quantities of interest can be easily calculated from these values. The integral equation presented here has the advantage that no higher-order derivatives of the kernel

function are involved. Also, since the fields in a region only depend on the surface current distribution on the region boundary, considerable computer time may be saved for problems with many regions. Another nice feature of this formulation is the  $A_1^s$  is proportional to the voltage drop along the conductor. Since the total current in the conductor is known, one may use it directly to calculate power loss and line impedance.

For the geometry of rotational symmetry where current flows only in the  $\phi$  -direction in the  $(\rho, \phi, z)$  coordinate system,

$\rho J^s$  is a constant for any given plane at  $\phi = \text{constant}$  in a conductor [2]. Hence, we may introduce a source component magnetic potential  $A^s = C / \rho$  where C is a constant. By using this potential, one can obtain a boundary integral equation similar to that described above in which C may be treated as an unknown.

### IMPLEMENTATION

The above boundary integral equation has been solved by using the Galerkin method [5,6]. Boundaries, i.e. the interfaces of materials are modelled exactly by using segments (lines, arcs, or splines). These segments are then divided into small sections which are referred to as boundary elements. The equivalent surface current distribution on each element is approximated by linear shape function [6]. When these discretized currents are used in (11) and (12) and the resulting equations tested with the same shape function on each element, a set of linear simultaneous equations for the unknowns of the surface equivalent current density coefficients and the source component potentials is obtained. The system of equations is then solved for the coefficients of the expansion function. Once the equivalent currents are determined, the potential, EM fields and induced currents can be calculated at any point by integration of the equivalent currents with the Green's functions. As well, parameters like forces, torques, ohmic loss, stored energy, and line impedance are readily calculated.

Using OERSTED on a PC or workstation, the geometry, material properties (conductivity, permeability, B-H curve for nonlinear materials), excitations (impressed current or impressed field), and operating frequency can be entered by using a keyboard and a mouse/digitizer through a menu-driven interface and are immediately displayed. This approach minimizes human error and the time required to enter or modify a particular problem. User may solve the problem interactively or use the BATCH command for unattended operation of the program. After the problem is solved, field value can be obtained at any desired location throughout the entire problem domain. Field distributions can be displayed in the form of contour plots, color maps, surface presentations, and graphs. Transverse field components can also be displayed in the form of arrow plots. The results can be saved in a data base for future use, and can also be printed out on hard copy.

### VERIFICATION AND APPLICATIONS

OERSTED has been tested for several simple geometries where analytic solutions are available; a circular conductor (2D) excited by surface or volume current and a spherical conductor (RS) excited by an impressed field. The results obtained by using OERSTED are accurate up to 5 digits for a wide frequency range. Several tests and applications are given below:

#### 1) A Three-Phase Bus System

The three-phase bus system is shown in Fig. 1. The conductors are made of copper tubes ( $\sigma = 5.8 \times 10^7 S/m$ ). The three phase currents are

$$I_1 -1 < 120^\circ >, I_2 -1 < 0^\circ >, I_3 -1 < -120^\circ > \text{ (kA)} \quad (15)$$

and the operating frequency is 400 Hz. In this problem, 10 elements have been used on each surface. The calculated voltage drops along the conductors are given in Table I as compared to the measured values [3].

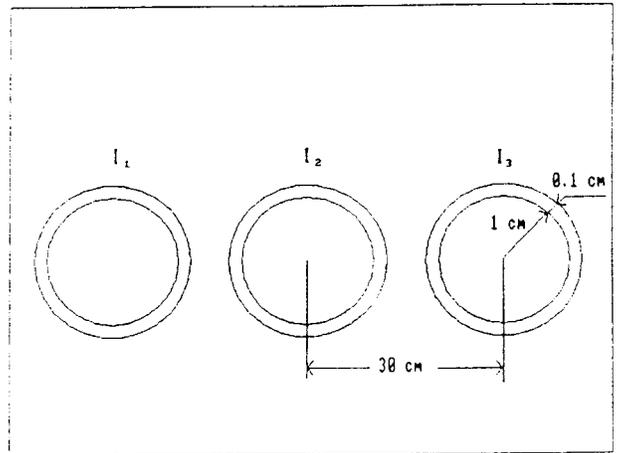


Fig. 1: Geometry of a three-phase bus system.

Table I. Voltage Drops Along Conductors

Methods	$V_1(V/m)$	$V_2(V/m)$	$V_3(V/m)$
Measurement	1.936	1.718	1.860
OERSTED	1.934	1.697	1.852

One observes an excellent agreement between OERSTED and measured results.

#### 2) A Pipe-Type Cable

A pipe-type cable is shown in Fig. 2 which consists of a pipe  $\sigma = 3.9077 \times 10^6 S/m, \mu_r = 500$  and three circular

conductors  $\sigma = 3.406 \times 10^7 S/m$ ,  $\mu_r = 1$ ). The unit of dimension is mm. The pipe is treated as the reference conductor. As shown in Fig. 2, 89 elements are used to approximate the equivalent currents. The impedances calculated by using OERSTED are listed in Tables II and III where the units for resistances and inductances are  $m\Omega/m$  and  $\mu H/m$ , respectively. This structure has been analyzed in [7] with a FEM, the results presented here are in good agreement with those in [7].

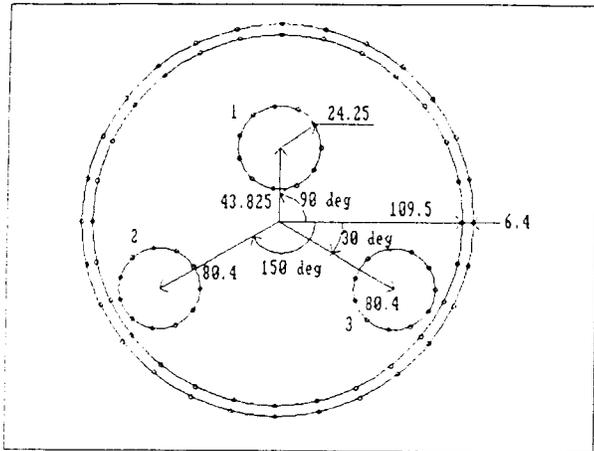


Fig. 2: Geometry of a pipe-type cable.

Table II. Resistances of a Pipe-Type Cable

f(Hz)	R <sub>11</sub>	R <sub>12</sub>	R <sub>22</sub>	R <sub>23</sub>
1	0.07288	0.05698	0.07290	0.05697
10	0.1108	0.0937	0.1117	0.0934
100	0.3600	0.3202	0.3753	0.3154
1000	1.194	0.9885	1.419	0.9284
10000	3.804	2.548	6.595	1.944
100000	5.731	3.400	12.28	2.190

Table III. Inductances of a Pipe-Type Cable

F(Hz)	L <sub>11</sub>	L <sub>12</sub>	L <sub>22</sub>	L <sub>23</sub>
1	2.327	1.913	2.441	1.838
10	1.976	1.569	2.082	1.496
100	0.8609	0.5059	0.9341	0.4457
1000	0.4656	0.1630	0.4975	0.1146

10000	0.3141	0.05355	0.2735	0.02295
100000	0.2850	0.03770	0.2010	0.01506

### 3) An Induction Heating System

The cross section of right hand side of the load of an induction heating system is shown in Fig. 3. This load consists of two turns of serially connected copper wire (work coil,  $\sigma = 5.8 \times 10^7 S/m$ ) which is in close proximity to the heated material (work piece,  $\sigma = 3.88 \times 10^7 S/m$ ). One important consideration in the coil design procedure is the impedance of the work coil and work piece which should be such as to allow for an efficient flow of energy from the source [8]. The system considered here was analyzed previously by using a FEM [8]. Three different load conditions are analyzed which correspond to the cases no heat ( $f=450$  kHz,  $d=0.015$  in), onset of heating ( $f=416$  kHz,  $d=0.045$  in) and efficient heating ( $f=384$  kHz,  $d=0.085$  in), respectively. The load impedances obtained by using OERSTED are shown in Table IV in which the units of resistance and inductance are  $\Omega$  and  $\mu H$ , respectively, and they are compared with the results of the FEM and measurement [8].

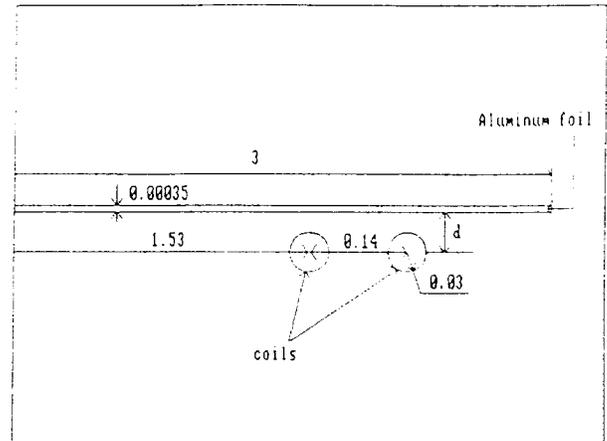


Fig. 3: Cross-section of a work coil-work piece at the symmetry plane.

Table IV. Impedance of Work Coil-Work Piece

F(kHz)	D(in)	OERSTED		FEM		Measured	
		R	L	R	L	R	L
450	0.015	0.297	0.149	0.269	0.153	0.344	0.189
416	0.045	0.222	0.216	0.202	0.222	0.249	0.269
384	0.085	0.168	0.282	0.155	0.289	0.185	0.343

Note that there is a quite large discrepancy between the computed and measured data. The main reason for this difference is that a rectangular work coil instead of a circular coil was used in the measurement. The magnetic potential contour plots for the above three conditions are given in Figs. 4-6. 321 elements have been used in analyzing this structure.

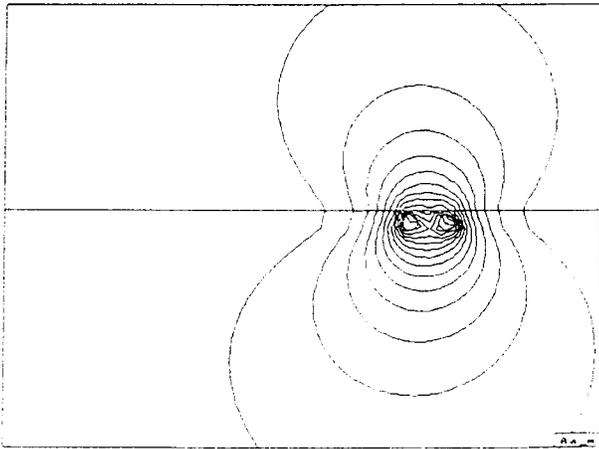


Fig. 4: Contour plot in the case of no heat.

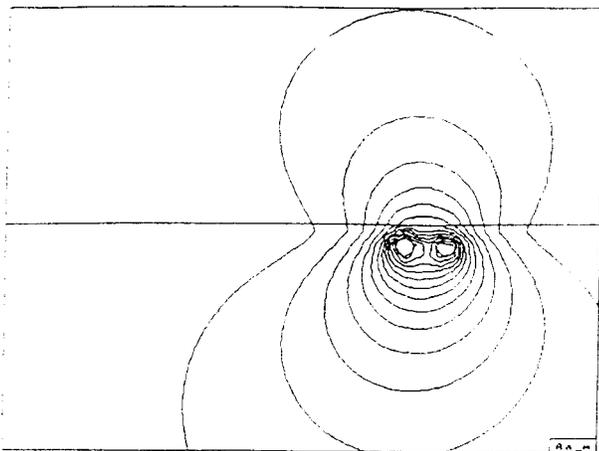


Fig. 5: Contour plot in the case of onset heating.

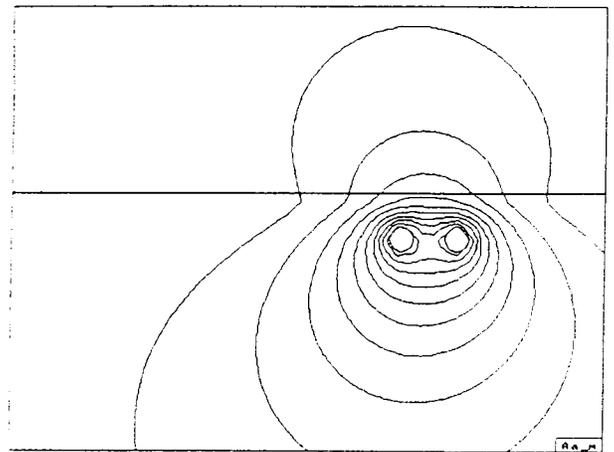


Fig. 6: Contour plot in the case of efficient heating.

#### 4) A Multilayer Coil

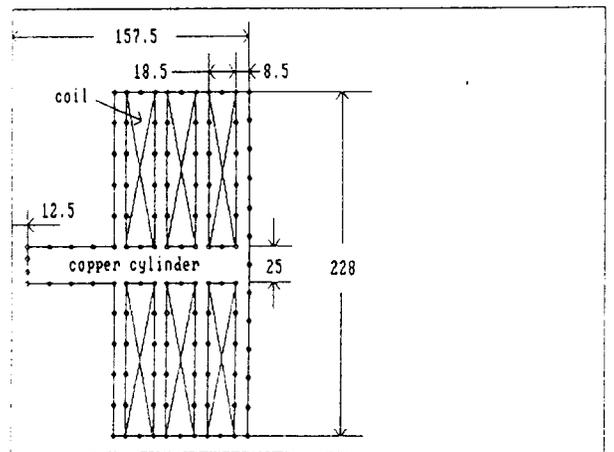


Fig. 7: Cross-section of a multilayer coil at the symmetry plane.

In Fig. 7, we show the cross-section of a multilayer eddy current type coil at the symmetry plane. This kind of coil can be used to generate AC high magnetic field. Due to the eddy current in the copper cylinder, high field will be produced in the hole which is the small area formed by the copper cylinder. 125 elements has been employed to model the coil, and the contour plot of calculated magnetic potential at  $f=60$  Hz is shown in Fig. 8. In this case, a total of 1 A current is assigned to each coil and a total of 0 A current is enforced in the copper cylinder. The magnetic flux density at the center of the hole is 0.15 mT.

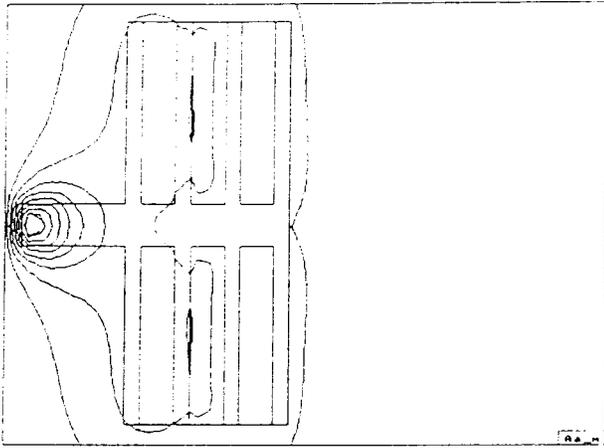


Fig. 8: Potential distribution in a coil.

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### CONCLUSIONS

A software package, OERSTED<sup>®</sup>, for simulating 2D/RS eddy current problem has been introduced in this paper. The boundary integral equation formulation and its numerical solution are described. Several application examples are presented to show the accuracy, efficiency and reliability of the program.

The BEM, combined with a highly interactive graphical user interface, makes OERSTED a very effective and powerful tool to simulate and analyze eddy current problem.

### ACKNOWLEDGEMENTS

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