

# Calculation of Transmission Line Parameters with the Boundary Element Method

## INTRODUCTION

The objective of this paper is to demonstrate the efficient calculation of design parameters using the boundary element method coupled with a highly interactive package for entering the transmission line configuration. The geometry of the problem that can be solved is arbitrary. Conductors may be infinitesimally thin or may be of finite area. As well, the conductors may be embedded in one or more dielectric materials.

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CALCULATION OF TRANSMISSION LINE PARAMETERS  
WITH THE BOUNDARY ELEMENT METHOD

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1. INTRODUCTION

The objective of this paper is to demonstrate the efficient calculation of design parameters using the boundary element method coupled with a highly interactive package for entering the transmission line configuration. The geometry of the problem that can be solved is arbitrary. Conductors may be infinitesimally thin or may be of finite area. As well, the conductors may be embedded in one or more dielectric materials.

With the advent of powerful microcomputers, computations that were once only possible on mini and mainframe computers, are now possible on microcomputers. In addition, microcomputers offer highly interactive graphics capabilities which can be an invaluable aid in the design of a system. The geometry, material properties, and other information can be entered with the help of a mouse or digitizer and immediately displayed. This approach minimizes human error and the time required to enter or modify a particular problem. Emphasis has been placed on the solution time of various problems, when in fact the data preparation can be by far the most time consuming. Data that takes hours to enter by hand can be entered in minutes with the help of a powerful user interface.

Calculation of design parameters for transmission line configurations, where a static approximation is adequate, requires the solution of Laplace's equation in two dimensions under prescribed boundary conditions. The boundary element method has been shown to be an efficient technique for the solution of Laplace's equation for piecewise homogeneous media [4]. This is, mainly, due to the reduction of one in dimensionality as all the unknowns are located only on the boundaries and interfaces. This differs from the finite difference and finite element methods in which the whole domain must be discretized. The unknown, computed using the boundary element method, is the equivalent charge that sustains the field. Once the equivalent charge is known any parameter can be derived.

A number of example problems are solved and compared with results available in the literature. All calculations were performed on an IBM-PC microcomputer.

2. PROBLEM DESCRIPTION AND FORMULATION

The problem consists of two or more conductors in a bounded or unbounded region containing one or more dielectrics. Figure 1 illustrates a general unbounded transmission line configuration with multiple dielectrics.

The capacitance matrix is found by setting the  $j^{\text{th}}$  conductor to one volt and all the other conductors to zero volts. The free charge per unit length on the  $i^{\text{th}}$  conductor is the  $ij^{\text{th}}$  element of the capacitance matrix.

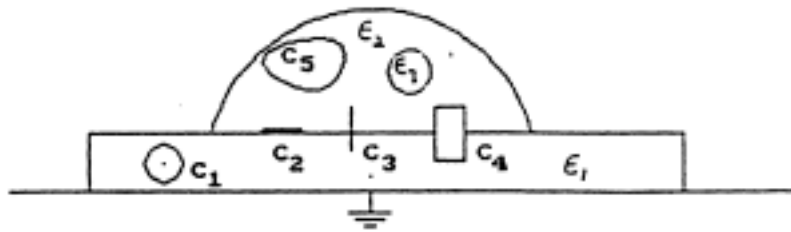


Fig. 1. A conductor-dielectric transmission line arrangement.

The inductance matrix can be found by taking the inverse of the capacitance matrix with the product of the free space permittivity and the free space permeability [1].

$$[L] = \mu_0 \epsilon_0 [C_0]^{-1} \quad (1)$$

$C_0$  is the free space capacitance matrix. The inductance calculation assumes all current is flowing on the surface of the conductor.

On the dielectric interfaces the equivalent charge is the polarization charge that would exist if the materials were replaced by free space and an equivalent charge placed on the interface [2]. The interface between conductors and dielectrics consists of a free charge plus a polarization charge forming the total equivalent charge.

The electric flux density at each point of a conductor interface is discontinuous by an amount proportional to the free charge. That is

$$\epsilon_2 \frac{d\phi_2}{dn} - \epsilon_1 \frac{d\phi_1}{dn} = -\epsilon_0 \quad (2)$$

where  $n$  is drawn from medium (1) into medium (2). For a conductor of finite cross section the interior field is zero yielding

$$\epsilon_2 \frac{d\phi_2}{dn} = -\epsilon_0 \quad (3)$$

where  $n$  is the outward normal and  $\epsilon_2$  is the material exterior to the conductor.

To calculate the free charge the equivalent charge must be found from which the normal derivative of the field may be calculated. The total free charge per unit length on the  $i^{\text{th}}$  conductor is simply

$$Q_i = \int \epsilon_0 (\underline{r}') dr' \quad (4)$$

where the integral is taken over the boundary of the  $i^{\text{th}}$  conductor. The charge is calculated with the  $j^{\text{th}}$  conductor at one volt. Thus

$$Q_i = \sum_{j=1}^{N_c} C_{i,j} V_j \quad i=1,2,\dots,N_c \quad (5)$$

or  $C_{ij} = Q_i$  when the  $j^{\text{th}}$  conductor is fixed at one volt.  $N_c$  is the total number of conductors less the ground or reference conductor.

The equivalent charge is found by forcing it to satisfy prescribed boundary conditions under a function which relates the location and effect of the charge to any point on the boundary or interfaces. This function is referred to as a Green's function or, more descriptively, the influence function.

The potential at any point in the region due to the equivalent charge is expressed as

$$\phi(\underline{r}) = \frac{1}{2\pi\epsilon_0} \int_{\partial b} \ln \frac{k}{|\underline{r}-\underline{r}'|} \sigma(\underline{r}') d\mathbf{r}' \quad (6)$$

where  $\epsilon_0$  is the free space permittivity and  $\sigma(\underline{r}')$  is the equivalent charge on the conductors and interfaces. The quantity  $k$  is any constant greater than the largest distance between any two points on the conductors and interfaces.

For the interfaces the continuity of the electric flux density yields

$$\epsilon_2 \frac{d\phi_2}{dn} - \epsilon_1 \frac{d\phi_1}{dn} = 0 \quad (7)$$

Substituting the integral form of the normal components of the electric field produces

$$(\epsilon_1 - \epsilon_2) \int_{\partial b} G'(\underline{r}, \underline{r}') \sigma(\underline{r}') d\mathbf{r}' + (\epsilon_1 + \epsilon_2) \sigma(\underline{r}) = 0 \quad (8)$$

where  $G'(\underline{r}, \underline{r}')$  is the normal derivative of the Green's function with respect to the unprimed variable and  $\partial b$  is all the interfaces and conductor boundaries.

Equation (6) is enforced on all conductor boundaries and equation (8) enforced on all interfaces to determine the equivalent charge [9].

### 3. BOUNDARY ELEMENT TECHNIQUE

To compute the equivalent charge, the coupled integral equations (6) and (8) are solved using the boundary element method (BEM). BEM is a variation of the method of moments. The conductors and interfaces are divided into smaller subdivisions, referred to as elements, over which the charge as well as the geometry is defined parametrically. The linear parametric representation of the charge over element  $l$  is

$$\sigma_l(\xi) = a_n \xi + a_m (1-\xi) \quad 0 \leq \xi \leq 1 \quad (9)$$

and  $a_n$  and  $a_m$  are the unknowns to be determined. Equation (9) can be put in vector form as

$$\underline{\sigma} = \underline{g}^T \underline{a} \quad (10)$$

where  $\underline{g}^T$  are the linear functions. The geometry over each element is modeled to a higher order to accommodate curvature. To accumulate the system matrix, Galerkin's method [3] is applied to either equation (6) or (8). For conductor boundaries the potential is enforced using equation (6).

Substituting (10) into (6) and applying Galerkin's method yields

$$\int_{\partial b} \underline{a}(\underline{r}) \phi(\underline{r}) = \int_{\partial b} \underline{a}(\underline{r}) \int_{\partial b \cup \partial i} G(\underline{r}, \underline{r}') \underline{a}_V(\underline{r}') d\underline{r}' \underline{a} \quad (11)$$

where  $\partial b$  contains all conductors and  $\partial i$  are all the interfaces.

For interfaces, equation (8) is enforced using Galerkin's method. This results in a system of equations which is diagonally strong provided the integrations are performed accurately. To insure this, special integration schemes are required when the charge and observation points coincide ( $\underline{r}=\underline{r}'$ ). The resulting system of equations is then solved yielding the desired equivalent charge. More in-depth information on the boundary element method can be found in reference [4].

#### 4. SOLVED PROBLEMS

Five examples are solved using the ELECTRO computer-aided engineering tool. The results agree well with those available in the references. All capacitance values are in farads per meter and the inductance values are in henrys per meter. Impedances are in ohms.

##### Example 1

The capacitance of two coupled microstrips is calculated. The geometry and dimensions of the problem are shown in Figure 2.

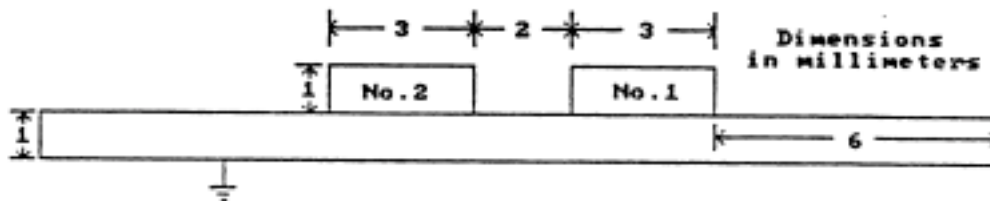


Fig. 2. Two coupled microstrips.

The dielectric above the finite ground plane has a relative permittivity of 2. Eight elements were used on the ground plane and two elements on each side of the two microstrips. Four elements are placed on the interface between the conductors and ten elements on the remainder of the interface (top and two sides). The solution time was three minutes. The results are compared with those of [7] in which an infinite ground plane and dielectric is assumed. Table I contains the results of both computations. Results are in agreement.

Table I  
Comparison of capacitance calculations

	ELECTRO	Reference [7]
$C_{11}$	$0.9088 \times 10^{-10}$	$0.9224 \times 10^{-10}$
$C_{12}$	$-0.8591 \times 10^{-11}$	$-0.8504 \times 10^{-11}$
$C_{21}$	$-0.8591 \times 10^{-11}$	$-0.8504 \times 10^{-11}$
$C_{22}$	$0.9088 \times 10^{-10}$	$0.9224 \times 10^{-10}$



### Example 2

Figure 3 shows three conductors in three different dielectric media. The problem was extracted from [1] and the results compared.

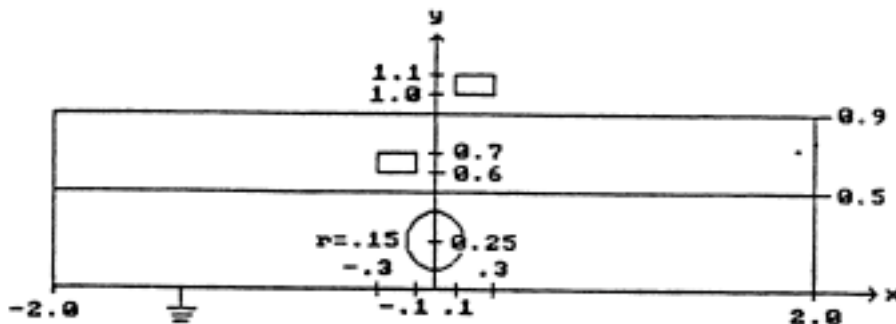


Fig. 3. Conductor arrangement for Example 2.

The center rectangle is conductor 1, the upper rectangle is conductor 2 and the circle is conductor 3. The relative dielectric constant of the upper and lower slab are 4.5 and 6.8, respectively. Eight elements were placed on all three conductors. Twelve elements were placed on each horizontal interface and the ground plane. One element was put on each vertical interface. Ten minutes was required to enter the problem and the solution time was ten minutes. The results in Table II are the capacitance with and without the materials present. The inductance values are also included. The results are seen to be in agreement.

Table II  
Comparison of results for Example 2

i	j	$C_{ij}$	$C_{0ij}$	$L_{ij}$
ELECTRO				
1	1	.1267 x 10 <sup>-9</sup>	.2874 x 10 <sup>-10</sup>	.4960 x 10 <sup>-8</sup>
1	2	-.1312 x 10 <sup>-10</sup>	-.7912 x 10 <sup>-11</sup>	.2094 x 10 <sup>-8</sup>
1	3	-.7187 x 10 <sup>-10</sup>	-.1251 x 10 <sup>-10</sup>	.1176 x 10 <sup>-8</sup>
2	1	-.1312 x 10 <sup>-10</sup>	-.7912 x 10 <sup>-11</sup>	.2094 x 10 <sup>-8</sup>
2	2	.3383 x 10 <sup>-10</sup>	.2056 x 10 <sup>-10</sup>	.6342 x 10 <sup>-8</sup>
2	3	-.7493 x 10 <sup>-11</sup>	-.3238 x 10 <sup>-11</sup>	.7984 x 10 <sup>-7</sup>
3	1	-.7187 x 10 <sup>-10</sup>	-.1251 x 10 <sup>-10</sup>	.1176 x 10 <sup>-8</sup>
3	2	-.7499 x 10 <sup>-11</sup>	-.3238 x 10 <sup>-11</sup>	.7984 x 10 <sup>-7</sup>
3	3	.3764 x 10 <sup>-9</sup>	.5854 x 10 <sup>-10</sup>	.2196 x 10 <sup>-8</sup>
Reference [1]				
1	1	.1244 x 10 <sup>-9</sup>	.2828 x 10 <sup>-10</sup>	.4965 x 10 <sup>-8</sup>
1	2	-.1300 x 10 <sup>-10</sup>	-.7678 x 10 <sup>-11</sup>	.1996 x 10 <sup>-8</sup>
1	3	-.8825 x 10 <sup>-10</sup>	-.1181 x 10 <sup>-10</sup>	.1183 x 10 <sup>-8</sup>
2	1	-.1300 x 10 <sup>-10</sup>	-.7678 x 10 <sup>-11</sup>	.1996 x 10 <sup>-8</sup>
2	2	.3340 x 10 <sup>-10</sup>	.2090 x 10 <sup>-10</sup>	.6163 x 10 <sup>-8</sup>
2	3	-.7196 x 10 <sup>-11</sup>	-.3030 x 10 <sup>-11</sup>	.7728 x 10 <sup>-7</sup>
3	1	-.8825 x 10 <sup>-10</sup>	-.1181 x 10 <sup>-10</sup>	.1183 x 10 <sup>-8</sup>
3	2	-.7196 x 10 <sup>-11</sup>	-.3030 x 10 <sup>-11</sup>	.7728 x 10 <sup>-7</sup>
3	3	.3523 x 10 <sup>-9</sup>	.5468 x 10 <sup>-10</sup>	.2331 x 10 <sup>-8</sup>

### Example 3

The impedance is calculated for a single microstripline of infinitesimal thickness touching a dielectric substrate, as shown in Figure 4.

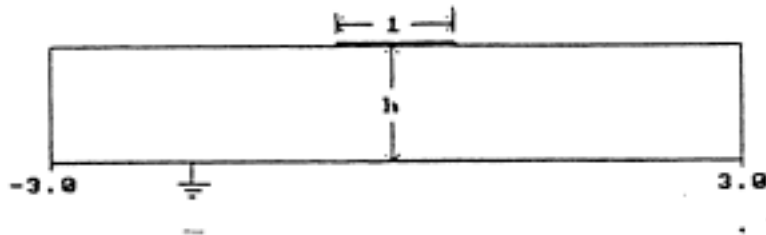


Fig. 4. Single microstripline.

The relative permittivity of the dielectric is 51. The impedance can be calculated from the capacitance matrix, free space capacitance matrix and velocity of light as follows.

$$Z_0 = \frac{1}{v_0(CC_0)^{1/2}} \quad (12)$$

Ten elements were placed on the strip, ten elements on each side of the strip and twenty elements on the ground plane. The results are compared with [5] where an infinite ground plane and dielectric is assumed. The results of the two methods are compared in Table III. The impedance values calculated by ELECTRO are slightly greater due to the finite dielectric and ground plane. Approximately two minutes was required to enter the problem and five minutes required for each calculation.

Table III  
Impedance for single microstrip.

l/h	ELECTRO	Reference [5]
10.0	4.484	4.407
5.00	7.925	7.802
2.50	13.14	12.93
1.00	22.58	22.14

#### Example 4

The even and odd mode capacitance ( $C_e, C_o$ ), impedance ( $Z_e, Z_o$ ) and effective relative permittivity ( $K_e, K_o$ ) are calculated for a coupled pair of microstrip transmission lines. The relative permittivity of the slab is 10. The even and odd mode values were derived from the one-by-one modes [8]. The geometry of the pair is as shown in Figure 5 and compared with the results in [6].

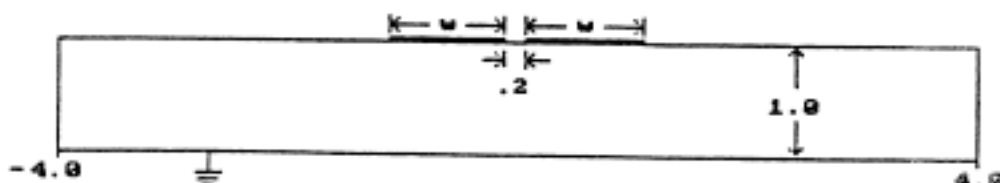


Fig. 5. Coupled pair of microstrip line.

Eight elements were placed on each strip, four elements on the interface between the strips, fourteen elements on the remainder of the top part of the dielectric, one on each side and fourteen elements on the lower part of the dielectric. Fourteen elements were placed on the ground plane. The results are listed in Table IV. Less than five minutes was required to solve the problem.

Table IV  
Even and odd mode parameter results.

	ELECTRO results	Reference [6]
$w=0.1$		
Ce	$0.5526 \times 10^{-10}$	$0.5451 \times 10^{-10}$
Co	$0.1243 \times 10^{-9}$	$0.1205 \times 10^{-9}$
Ze	151.9	153.0
Zo	63.02	64.92
Ke	6.331	6.251
Ko	5.513	5.499
$w=0.5$		
Ce	$0.9605 \times 10^{-10}$	$0.9601 \times 10^{-10}$
Co	$0.2010 \times 10^{-9}$	$0.2010 \times 10^{-9}$
Ze	91.13	90.40
Zo	39.21	36.92
Ke	6.885	6.770
Ko	5.582	5.567
$w=1.0$		
Ce	$0.1413 \times 10^{-9}$	$0.1418 \times 10^{-9}$
Co	$0.2534 \times 10^{-9}$	$0.2569 \times 10^{-9}$
Ze	64.09	63.18
Zo	31.47	31.00
Ke	7.371	7.210
Ko	5.715	5.699

### Example 5

The even and odd mode capacitances are calculated for the configuration illustrated in Figure 6.

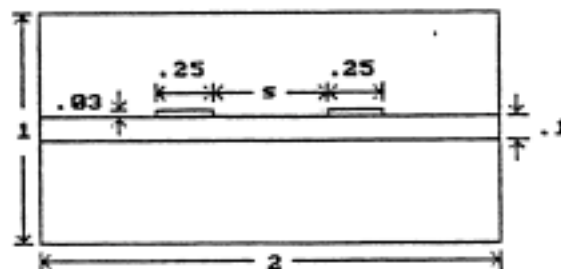


Fig. 6. Coupled suspended stripline.

The two conductors are placed on a dielectric with a relative constant of 10. Eight elements are contained on each stripline and sixteen on the outer conductor. Eight elements are located on the lower part of the dielectric, four between the conductors, and eight on the remaining. The total time to compute the parameters was eight minutes. Entering the problem required ten minutes. Results are listed in Table V.



Table V  
Parameters of coupled microstrip.

	Even Mode	Odd Mode
$s=0.5$		
C	$0.4316 \times 10^{-10}$	$0.7253 \times 10^{-10}$
Z	102.4	72.03
K	1.757	2.453
$s=0.2$		
C	$0.3567 \times 10^{-10}$	$0.1039 \times 10^{-9}$
Z	120.7	54.39
K	1.667	2.870
$s=0.1$		
C	$0.3325 \times 10^{-10}$	$0.1411 \times 10^{-9}$
Z	129.7	42.09
K	1.673	3.170

## 5. REMARKS

The boundary element technique, combined with a highly interactive user interface, automates the design of standard and non-standard transmission line configurations. Problem geometries, and materials can be conveniently described from the conceptual stage and be analyzed to obtain the desired design parameter. Accurate and reliable results are due to the boundary element technique and the user interface which minimizes human error.

## 6. REFERENCES

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