

# MAGNETIC ANALYSIS OF BRUSHLESS DC MOTORS USING THE BOUNDARY ELEMENT METHOD

## ABSTRACT

The advent of high speed microcomputers and available software has made possible the calculation of magnetic fields and torques of brushless DC motors. In particular, the Boundary Element Method (BEM) has shown to be an efficient tool for the design of these motors. This paper explains the BEM approach and addresses the key concerns facing the design engineer using such software, i.e. the ease of use, accuracy of solution, and the computation time required to solve the problem. A sample analysis is illustrated and a torque vs. angle curve is generated for a geometrical configuration to demonstrate the use of the BEM software in the design of brushless DC motors.

### Integrated Engineering Software - Website Links

[Home](#)[Products](#)[Support](#)[Technical Papers](#)

*"Page Down" or use scroll bars to read the article*



## **MAGNETIC ANALYSIS OF BRUSHLESS DC MOTORS USING THE BOUNDARY ELEMENT METHOD**

Y. Bulent Yildir, Bruce W. Klimpke, K.M. Prasad

Integrated Engineering Software Inc.  
347-435 Ellice Avenue, Winnipeg, Manitoba Canada R3B 1Y6

### **ABSTRACT**

The advent of high speed microcomputers and available software has made possible the calculation of magnetic fields and torques of brushless DC motors. In particular, the Boundary Element Method (BEM) has shown to be an efficient tool for the design of these motors. This paper explains the BEM approach and addresses the key concerns facing the design engineer using such software, i.e. the ease of use, accuracy of solution, and the computation time required to solve the problem. A sample analysis is illustrated and a torque vs. angle curve is generated for a geometrical configuration to demonstrate the use of the BEM software in the design of brushless DC motors.

### **INTRODUCTION**

Today's highly competitive international economy demands that companies search for new methods to increase efficiency, improve productivity and streamline operations. In the manufacturing sector, the reduction of costly prototype development and production represents a significant competitive advantage.

Magnetic field analysis for optimum design is now entering the mainstream of Computer-Aided Engineering. Field simulation permits electrical engineers to vastly improve both their visualization and manipulation of the electrical and magnetic fields which underlie all electrical products.

The purpose of field simulation is to create reliable and accurate prototypes using a computer. A computer graphics interface allows the engineer to draw a geometric model, specify the material properties of objects and to identify the source of electromagnetic fields. The simulator then computes the fields that exist throughout the structure and allows the solution to be both quantitatively manipulated and displayed on the screen.

Accuracy in the calculation of field distributions and device parameters remains the foremost consideration in the choice of the appropriate tools, followed by speed of solution and cost. Over the last few decades numerical methods have gained in accuracy and applicability. The growing power of the microcomputer has put the final touch on the appeal of the computer-aided approach.

Two distinct approaches exist for the solution for field problems: domain-type and boundary-type. Domain-type formulation is the direct solution of the differential equation governing the field. The finite difference (FDM) and finite element (FEM) methods are the two most commonly used domain-type methods.

In the finite difference approach, the differential operator is discretized, utilizing a truncated Taylor series expansion in each coordinate direction, and applied at each point of a rectilinear grid placed on the problem region. The method usually involves an iterative process [1].

The finite element method uses a variational technique in which the potential is approximated by a sequence of functions defined over the entire domain of the problem. Relating these approximations to the

operator equation, through minimizing a functional that is proportional to the energy of the system, yields the nodal values of the solution [2].

The second approach to the solution of field problems is the boundary integral equation formulation on which the Boundary Element Method (BEM) is based.

## BOUNDARY ELEMENT METHOD

The boundary element method is a variation of the moment methods [9]. It is based on the integral equation formulation of the fields and defines unknowns only on boundaries. This leads to a reduction of complexity in modelling and solving the problem. The dimensionality of the problem is reduced by one without sacrificing the accuracy. In fact, the accuracy obtained is superior to that of the finite element method [3].

The input data required by the boundary element method is much less than that needed by the finite difference and finite element methods. No fictitious truncation of the domain is needed for open region problems. Also, the accuracy of the problem definition is invariant to its geometric complexity [5].

Enforcing prescribed boundary conditions, an equivalent source that would sustain the field is found with the use of a free space Green's function. The effect of the equivalent source at any point on the boundary is found from the source location and Green's function. The use of Green's function effectively eliminates the need for a finite element mesh or a finite difference grid.

Computation of the potential and field at an arbitrary point involves direct integration of the equivalent source thus obtained. Since no interpolation is involved in these computations, the results obtained are more stable than those of the finite element method.

In addition, the boundary element method provides very accurate results, as all parameters are calculated by integrating the equivalent source. In finite elements, fields are calculated by numerically differentiating the potential, which may give rise to erroneous results as numerical differentiation introduces discontinuities.

In the boundary element method, potentials and fields can be obtained for every observation point in a region that extends from the near to the far zone, with the same degree of accuracy. Therefore, it is possible to zoom into any area and obtain accurate results. In the FEM, if one were to zoom into a single element, one would only obtain interpolations from the nodal values of that element.

Force and torque calculations in the boundary element method do not require solutions of the problem with relative displacements. Integration of the cross product of the equivalent source and the magnetic induction over a region of interest yields the force acting on that region. Torque calculations are performed with the same ease and accuracy as force calculations.

## MAGNETIC FIELD EQUATIONS

Maxwell's equations interrelate the electromagnetic field quantities, sources and materials in a problem. Imposing proper boundary conditions, a partial differential equation which governs the fields can be formulated. This general equation can be simplified by utilizing the problem's symmetry. Usually the geometry of the problem determines whether a 2D or 3D analysis is suitable. In this paper, we will restrict our attention to 2D static and quasi-static magnetic fields.

From Maxwell's equations we have,

$$\nabla \cdot \underline{B} = 0 \tag{1}$$

and

$$\nabla \times \underline{H} = \underline{J} \quad (2)$$

A magnetic vector potential which satisfies (1) can be defined as

$$\underline{B} = \nabla \times \underline{A} \quad (3)$$

In an isotropic material B and H are related by

$$\underline{B} = \mu(B)\underline{H} \quad (4)$$

Taking curl on both sides and simplifying the right hand side with elementary vector analysis provides

$$\nabla \times \underline{B} = \nabla \mu(B) \times \underline{H} + \mu(B) \nabla \times \underline{H} \quad (5)$$

Substitution of (2) and (3) into (5) yields

$$\nabla \times \nabla \times \underline{A} = \nabla \mu(B) \times \underline{H} + \mu(B) \underline{J} \quad (6)$$

If  $\mu$  is independent of B (6) reduces to

$$\nabla \times \nabla \times \underline{A} = \mu \underline{J} \quad (7)$$

For the x-y two dimensional system (7) is simplified to

$$\nabla^2 A_z = \mu J_z \quad (8)$$

which is Poisson's equation. Because it is a uni-dimensional equation the subscript z for A and J is omitted hereafter.

In (8), if J is known, the solution for A is given from Green's theorem as

$$A(r) = \frac{\mu_0}{2\pi} \int G(r, r') J(r, r') dS' \quad (9)$$

Expression for 2D free space Green's function is given by

$$G(r, r') = \ln \frac{C}{|r - r'|} \quad (10)$$

where C is a constant chosen to be larger than the maximum possible dimension in the problem. When different permeable media exist in the problem, their effects can be accounted for by an equivalent current or magnetization that would sustain the field on the interfaces. The magnetic potential due to the permeable media is given by replacing J with the equivalent current density K in (9). To determine K, the interface equation for the continuity of the tangential component of H is enforced, that is

$$H_{t1} = H_{t2} \quad (11)$$

or

$$\frac{B_{t1}}{\mu_1} = \frac{B_{t2}}{\mu_2} \quad (12)$$

Substituting (9) in (3) with K replacing J and specializing for 2D gives the tangential component of B on the interface as

$$B_{t12} = \frac{\mu_o}{2\pi} \int \frac{\partial G}{\partial n}(r, r') K dS' \pm \frac{K}{2} \quad (13)$$

where  $\partial G / \partial n$  is the partial derivative of Green's function along the normal directed from region 1 to region 2 [6].

Substitution of (13) in (12) gives the integral interface equation,

$$(\mu_1 - \mu_2) \frac{\mu_o}{2\pi} \int \frac{\partial G}{\partial n}(r, r') K dS' = (\mu_1 + \mu_2) \frac{K}{2} \quad (14)$$

in which K is the only unknown. To determine K, boundaries and interfaces are divided into small sections which are referred to as boundary elements. The integral equation (14) is discretized along these elements using the Galerkin method to produce a set of linear simultaneous equations for the unknown equivalent current density coefficients [5], [7], [9]. The solution to this system of equations yields the equivalent current density distribution. Iterative implementation of this method provides solution to a non-linear problem. Impressed fields and contributions from permanent magnets form part of the forcing function in the integral equation formulation.

#### A SAMPLE PROBLEM

A four pole, twelve tooth, brushless permanent magnet motor is modeled, solved and the results analyzed. The rotor consists of four magnets uniformly magnetized in alternating directions. The coercivity of the magnets is 10100 Oersted and the remanent value is 10800 Gauss. The iron is cold-rolled carbon steel. The ampere-turns in each coil is set at 500 for the sample analysis. The cross-section of the motor geometry is illustrated in Figure 1. The dimensions of some of the parts are shown along with the direction in which the magnets are magnetized.

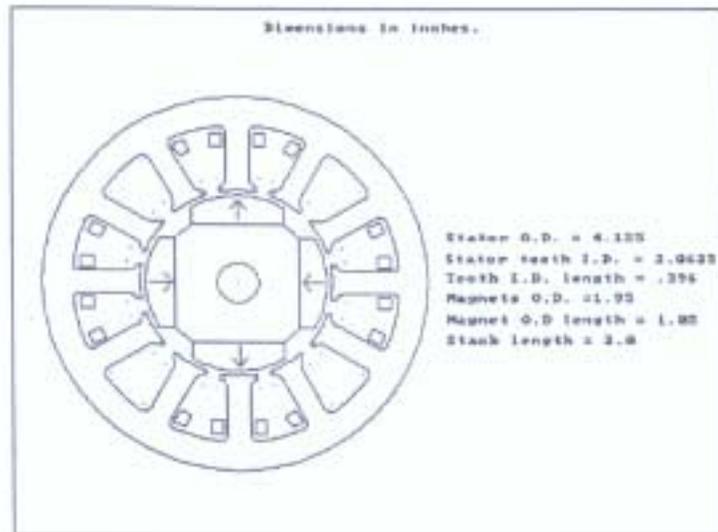


Figure 1. Dimensions and magnet orientations.

### MODELLING THE PROBLEM

To model the motor using MAGNETO the required input is the geometry, material properties (B-H curves and direction of the permanent magnets), and the currents in the windings. The problem boundaries are then discretized into subdivisions referred to as boundary elements. Areas where the material is rapidly being saturated require subdivisions known as subareas.

When initially modelling a problem, the most difficult task is entering the geometry of the problem. However, the advent of elaborate geometric modellers such as AUTOCAD or the one built into MAGNETO has greatly simplified this process. Geometry data can be readily transferred between more advanced CAD packages using the Initial Graphics Exchange Standard (IGES). The geometric modelling difficulty has thus been substantially reduced. As an illustration consider modelling the geometry in Figure 1. First the one tooth is modeled using the line and arc commands in MAGNETO as shown in Figure 2.

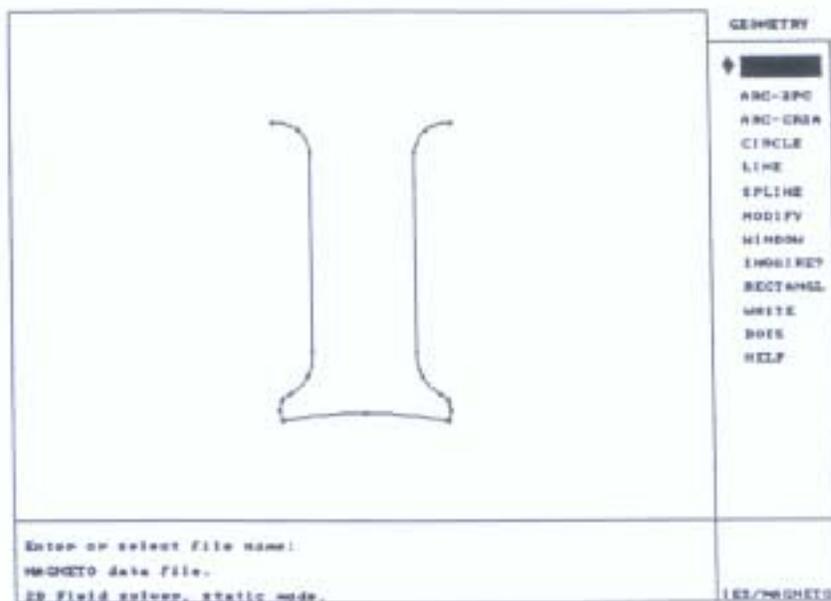


Figure 2. Initial tooth of stator.

The subsequent 11 teeth are then created by rotating and copying the tooth already created. The stator is completed by creating a circle on the outer periphery and placing the geometry of the coils in one slot and rotating and copying them to the other slots. The rotor magnets and enclosed iron are created in a similar fashion.

If the geometry of the problem had been previously created in a different CAD package that supported IGES, the entire structure could have been retrieved into MAGNETO in seconds.

The B-H curves of the irons and magnets can be specified next. These curves can be entered into the materials table if they do not exist already. For example, twelve points were extracted from a manufactures B-H curve for cold-rolled steel. The points were entered and the curve created as shown in Figure 3.

The magnets used are simply entered by specifying the coercive and remanent values. Once the material curves have been entered in the table they can be assigned to the rotor and stator iron and the magnets. The directions in which the magnets are magnetized are then entered to complete the material specification.

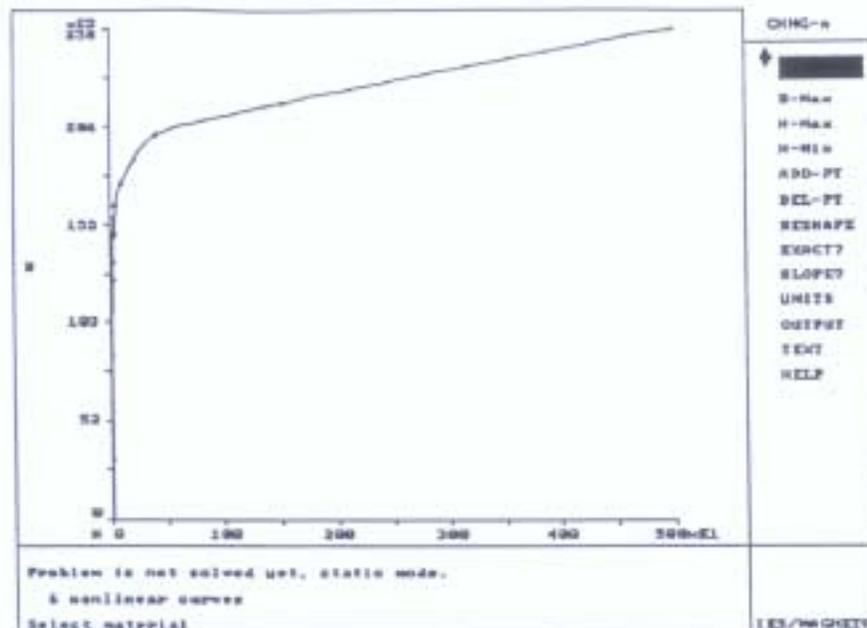


Figure 3. B-H curve of cold-rolled carbon steel.

The coil currents (Amp\*turns) are then assigned to the coils to complete the physical definition of the problem.

In order to solve the problem, boundary elements are required on all the iron and permanent magnet boundaries. That is, the geometry segments must be discretized into smaller subdivisions called boundary elements. The finer the discretization the more accurate the solution. Finer discretization, however, leads to longer solution time. The required boundary element density is determined by some basic guidelines and element optimization is subsequently achieved with experience. Areas where the iron is rapidly changing from an unsaturated to a saturated state may require that the area be divided into smaller areas called subareas. These are typically required near the teeth tips where the material properties are changing rapidly. Subareas have no effect for linear materials, only a small effect when the fields in a material are slowly varying. Upon discretization, the problem is ready to be solved.

## ANALYSIS AND RESULTS

Visual results are readily available to indicate design performance such as regions where the materials are saturating, flux density in the air gap and the overall field lines. Figure 4 illustrates the overall field lines and Figure 5 is a graph of the flux density crossing the middle of the air gap. The graph is calculated along a curve spanning 90 degrees above the top magnet. Integrating along the graph of Figure 5 would calculate the total flux crossing the gap.

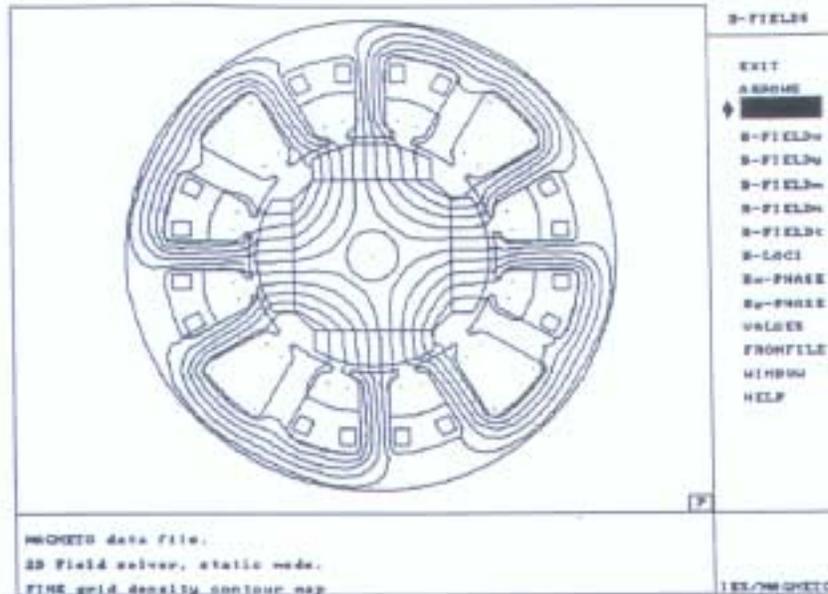


Figure 4. Field lines about motor.

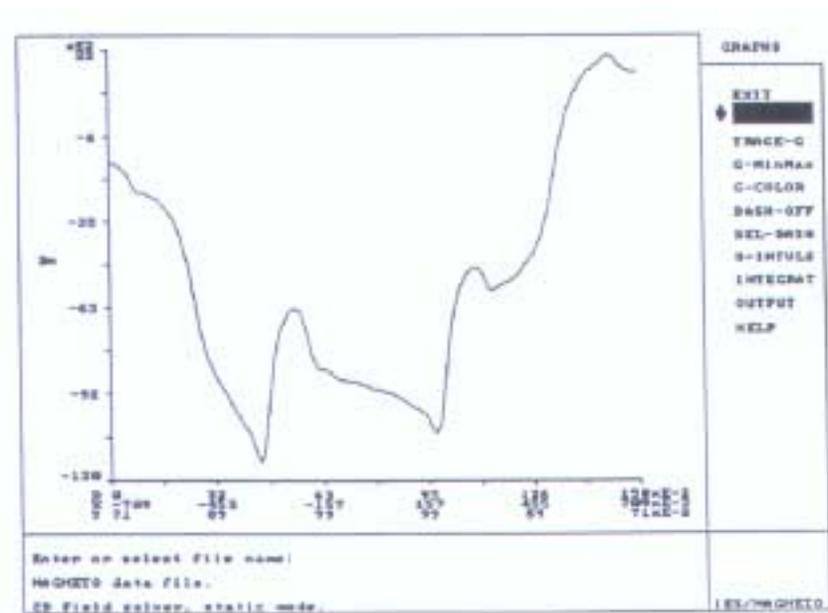


Figure 5. Flux density crossing air gap.

Often the most important parameter is the torque for a given rotor position and current. To obtain curves of torque versus rotor position for a specified current, an interactive batch facility is used. For example, the current is fixed at a given value. The rotor is then rotated a 5 degree increments for 45 degrees and stored in the database. As the BEM is used the discretization is unaffected for each rotor position and no re-meshing is required as it would be in the finite element method. A procedure is then followed to retrieve each problem from the database and calculate the torque. The actual calculations are not performed at this time but at a later time specified by the user, such as at lunch time or overnight. Such a procedure minimizes the user's time and optimizes the computer's use. The torque angle curve in Figure 6 was generated by this method. Angle 0 degrees corresponds to the rotor position in Figure 4 and angle 45 degrees corresponds to the rotor rotated 45 degrees counterclockwise to Figure 4.

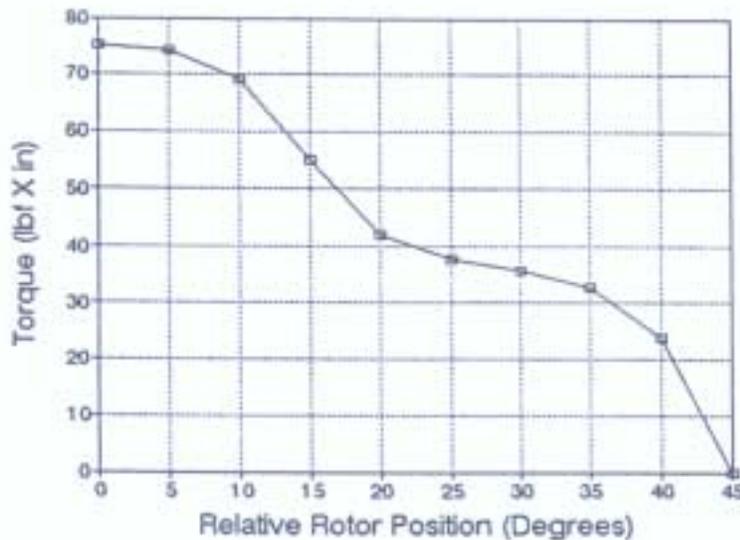


Figure 6. Torque vs. angle curve for 45 degree span.

Making use of the interactive batch facility enables the designer to try various geometries, materials and current levels to find an optimal design.

## CONCLUSIONS

The boundary element method has been shown to be an efficient technique for the simulation of electromagnetic design problems on a personal computer.

The main advantages of this method are the reduction of one in problem dimensionality, accurate modelling of geometry, elimination of differentiation and interpolation to calculate potential or its derivatives, precise results due to the smoothness of the integral operator [4] and sound means for checking the accuracy of the solution.

Inductances are calculated from the energy definition. Other parameters are calculated by integrating the equivalent current.

The boundary element method, combined with a highly interactive user interface on a personal computer, automates the analysis of electromagnetic field distributions and provides accuracy, speed and ease of use.

Problem geometries, materials and boundary conditions can be conveniently described from the conceptual stage and analyzed to obtain the optimum design.

## ACKNOWLEDGEMENTS

We acknowledge the invaluable contributions of our colleagues at Integrated Engineering Software Inc.

We are thankful to Mr. Jim Hendershott of MAGNA PHYSICS Corporation who supplied us with the motor configuration illustrated in the sample analysis section in this paper.

The development of the two-dimensional magnetic field analysis package MAGNETO was funded in part by an IRAP-M grant from the National Research Council of Canada.

## REFERENCES

1. G.E. Forsythe and W.R. Wasow, Finite Difference Methods for Partial Differential Equations. New York: John Wiley & Sons, 1960.
2. O.C. Zienkiewicz, The Finite Element Method in Engineering Science. New York: McGraw-Hill, 1971.
3. Y.B. Yildir, "Computer-Aided Field Analysis fo High Voltage Apparatus Using the Boundary Element", Proc. International Coil Winding Conference, Rosemont, Illinois, Oct. 5-8, 1987.
4. S.G. Mikhlin, Integral Equations, New York: Pergamon Press, 1864.
5. Y.B. Yildir, A Boundary Element Method for the Solution of Laplace's Equation in Three-Dimensional Space. Ph.D. Dissertation, University of Manitoba, Winnipeg, Canada R3T 2N2, 1985.
6. Stratton, J.A. Electromagnetic Theory. McGraw Hill Book Co., New York, 1941.
7. B.W. Klimpke, A Two-dimensional Multi-media Boundary Element Method. M.Sc. Dissertation, University of Manitoba, Winnipeg, Man. Canada R3T 2N2, 1983.
8. M.H. Lean and D.S. Bloomberg, "Nonlinear Boundary Element Method for Two-dimensional Magnetostatics", J. Appl. Phys., Vol 55, No.6, 1984.
9. R.F. Harrington, Field Computation by Moment Methods, Reprinted by R.F. Harrington, R.D. 2, West Lake Road, Cazenovia, N.Y. 1968.