PC-Based Electrostatic Field Calculation Techniques

ABSTRACT

When working with high voltage equipment, it is helpful to be able to predict what the electrostatic potentials are in the various regions of the equipment during operation. With the development of powerful microcomputers, it has become possible to perform electrostatic calculations of moderate complexity. Though performed at slower speeds than with mainframe computeres, these calculations provide comparable accuracy at a lower cost and with enhanced user interface capabilities. The Personal Computer (PC) based codes evaluated are commercially available and use the Boundary Element Method (BEM) and the Finite Element Method (FEM) to solve electrostatic field problems. These methods use derivative (FEM) and integral (BEM) techniques to solve Poisson's equation for the electrostatic potentials within the region of interest. Both codes allow the user to define the geometry of the problem as well as any dielectric materials with data entry devices (mouse, digitizing table...). The advantages of each method are described and results are shown. These results demonstrate the usefulness of PC-based codes for performing electrostatic analyses.

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Presented at the 12th Annual Ideas in Science and Electronics Exposition/Symposium, May 10, 1991.

PC-Based Electrostatic Field Calculation Techniques

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Abstract

When working with high voltage equipment, it is helpful to be able to predict what the electrostatic potentials are in the various regions of the equipment during operation. With the development of powerful microcomputers, it has become possible to perform electrostatic calculations of moderate complexity. Though performed at slower speeds than with mainframe computers, these calculations provide comparable accuracy at a lower cost and with enhanced user interface capabilities. The Personal Computer (PC) based codes evaluated are commercially available and use the Boundary-Element Method (BEM) and the Finite Element Method (FEM) to solve electrostatic field problems. These methods use derivative (FEM) and integral (BEM) techniques to solve Poisson's equation for the electrostatic potentials within the region of interest. Both codes allow the user to define the geometry of the problem as well as any dielectric materials with data entry devices (mouse, digitizing tablet ...). The advantages of each method are described and results are shown. These results demonstrate the usefulness of PC-based codes for performing electrostatic analyses.

I. INTRODUCTION

In determining solutions to electrostatic problems with a computer, Maxwell's equations are not solved in closed form, but the region of interest is divided up into small areas, a process called discretization. There are three approaches to solving the discretized Maxwell Equations : the Boundary Element Method (BEM), the Finite Element Method (FEM) and the Finite Difference Method (FDM) [1]. The BEM expresses Maxwell's equations in integral form and solves for the source of the field on the boundaries of the device. The FEM requires that the device geometry be broken up into a mesh of many small pieces of standard shape and expresses Maxwell's equations in differential form. The FDM also solves the equations in differential form, but in this case can solve them only with a regular mesh while the FEM allows irregularly shaped meshes. Only the BEM and FEM will be discussed.

II. FINITE ELEMENT METHOD

The FEM solves electrostatics problems by breaking the entire domain of interest into small triangular regions (called elements) as shown in Figure 1. The elements can be uniformly distributed or concentrated in highly divergent field areas. A solution for Laplace's equation ($\nabla^2 V=0$) is then found by minimizing the energy contained within each element [2]. The total energy of an element is given by :

$$W = \int \varepsilon/2\{[dV/dx]^2+[dV/dy]^2\}d\gamma \quad (1)$$

where V is the electrostatic potential, $d\gamma$ is the infinitesimal area element and ε is the dielectric constant.

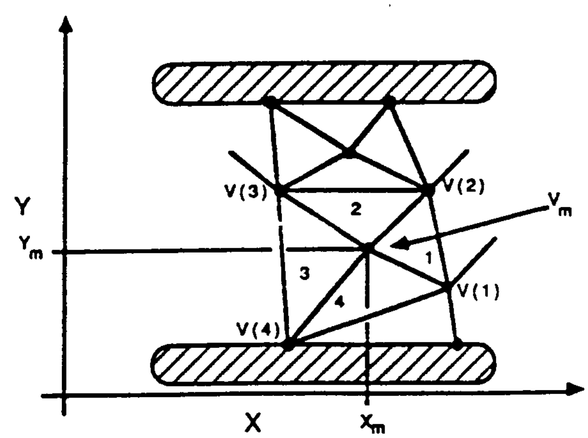

Fig 1. Example of a FEM Mesh

A first order FEM algorithm assumes that the potential varies linearly within the triangular elements. Therefore,

$$V(x,y) = \alpha_1^i + \alpha_2^i x + \alpha_3^i y \quad (2)$$

where α_1^i, α_2^i, and α_3^i are constants within the i'th element (see Figure 1). Equation 2 is then substituted into Equation 1 giving :

$$W^i = \varepsilon \gamma^i \{[\alpha_2^i]^2 + [\alpha_3^i]^2\}/2 \quad (3)$$

where γ is the area (for a two dimensional problem) of the triangular element. The α terms (as will be shown later) are a function of V_m.

The next step is to calculate the potential V_m shown in Figure 1. Values for α can be calculated as a function of V_m since the potentials at nodes 1-4 are known. This is accomplished by using Equation 2 as follows :

$$\begin{aligned} V(1) &= \alpha_1^1 + \alpha_2^1 x_1 + \alpha_3^1 y_1 \\ V(2) &= \alpha_1^1 + \alpha_2^1 x_2 + \alpha_3^1 y_2 \quad (4) \\ Vm &= \alpha_1^1 + \alpha_2^1 x_m + \alpha_3^1 y_m \end{aligned}$$

These equations can be solved for $\alpha_1^1(V_m)$, $\alpha_2^1(V_m)$, and $\alpha_3^1(V_m)$. The process is then repeated for each of the three remaining elements (2, 3, and 4).

The potential at (x_m, y_m) is then found by minimizing the total energy contained in the four surrounding elements. In mathematical terms, the following minimization equation is solved :

$$dW/dVm = 0 = (d/dVm)\ W^1 + (d/dVm)\ W^2 + (d/dVm)\ W^3 + (d/dVm)\ W^4 \quad (5)$$

The result is an approximation for the electrostatic potential V_m at the point (x_m, y_m). The distribution of the potentials is then given by Equation (2) within each element.

III. BOUNDARY ELEMENT METHOD

The BEM takes a different approach to solving electric field equations. As shown in Figure 2, the "elements" are placed only on the boundaries of the geometry. The elements are linear in contrast to the triangular elements used for the FEM. The BEM uses an integral formulation of Laplace's equation. Through the use of Green's theorem, the FEM type mesh is eliminated. A BEM solution can be completed in one of many ways. Due to the complexities involved, only a limited general approach will be discussed here [3].

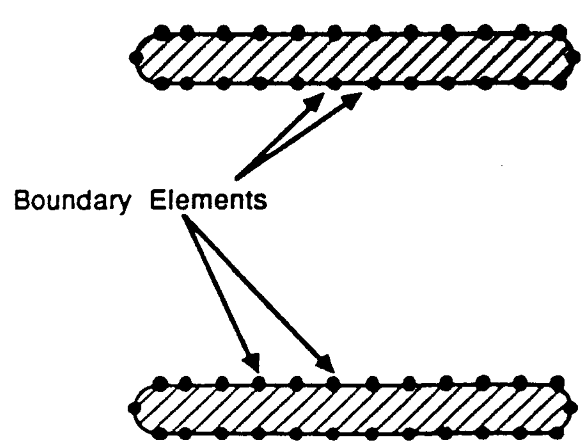

Fig 2. Example of BEM Element Placement

The first step in obtaining a BEM solution is to formulate an integral equation that describes the electrostatic potentials. The format of such an equation is as follows:

$$L\sigma = g \tag{6}$$

where L is a linear integral operator, g is known and σ is expanded as was done for the FEM (Equation 2) as follows:

$$\sigma = \Sigma_n \alpha_n \beta_n \tag{7}$$

where α_n are constant coefficients and the β_n terms are the expansion function. These functions ultimately determine the accuracy of the solution and therefore must be chosen very carefully.

The substitution of Equation (7) into Equation (6) (while assuming that L is linear) results in,

$$\Sigma_n \alpha_n L(\beta_n) = g \tag{8}$$

It is now required that a suitable "inner product" $\langle f,g \rangle$ be determined. The inner product must satisfy the following conditions [4],

$$\langle f,g \rangle = \langle g,f \rangle$$

$$\langle \alpha f+\beta g,h \rangle = \alpha\langle f,h \rangle + \beta\langle g,h \rangle$$

$$\langle f^*,f \rangle > 0 \quad \text{if } f \neq 0$$

$$\langle f^*,f \rangle = 0 \quad \text{if } f = 0$$

Using the Galerkin method [4], the inner product of each side of Equation (8) with β_m , the following equation is arrived at:

$$\Sigma_n \alpha_n \langle \beta_m, L\beta_n \rangle = \langle \beta_m ,g \rangle \tag{9}$$

This equation can be rewritten using matrix notation as,

$$S\alpha = b$$

where $S_{mn} = \langle \beta_m, L\beta_n \rangle$ and $b_m = \langle \beta_n, g \rangle$. The equation is then solved for the α vector. These coefficients along with Equation (7) define σ, which is the parameter of interest.

IV. COMPARISON OF BEM, FEM

The similarity between the BEM and FEM methods is minimal with advantages appearing for both calculation techniques. The first order FEM method results in equipotential lines that are linear within the elemental area. Discontinuities will occur at the boundaries of each element which may introduce significant error in highly divergent field regions. This difficulty associated with the FEM method can be minimized through the use of a higher order algorithm (modification of equation 2) or by increasing the number of elements. The number of elements is limited by the computer's available memory. The BEM method provides continuous equipotential contours that are free from discontinuities since a single potential function is calculated for the entire region of interest.

The BEM method provides an error check method that is inherent to the calculation of a potential function. By using the potential function to calculate the voltages at the defined boundaries, the accuracy of the BEM calculation can be tested. The calculated values should match the defined values. The FEM does not provide such a capability other than the minimization of discontinuities as discussed above.

The BEM is much more complex to program than the FEM. The BEM method requires complex integration techniques where the FEM utilizes linear algebra to solve for the fields. The BEM inherently solves open field problems where the entire region must be bounded for the FEM. Finite element methods require more data storage area due to the larger number of elements needed to complete accurate calculations. The BEM elements are confined to the geometrical boundaries of the problem.

V. SOFTWARE CAPABILITIES

To demonstrate the capabilities of PC based electrostatic calculation techniques, an electrostatic package which uses the BEM was evaluated. The ELECTRO [5] analysis code package allows the user to enter geometries using a mouse or a digitizing tablet. Also, the package provides graphics routines to present the data in a variety of formats. This includes equipotential contour plots , E-field plots, and 2 and 3 dimensional graphs of E-field values as a function of position. The program is also capable of calculating capacitance values and transmission line impedances. In order to demonstrate some of the capabilities, a sample problem will be solved and shown along with the results of several other simple examples.

A. Sample Problem

The Problem to be solved is shown in Figure 3. First the geometry is entered using one of the data entry

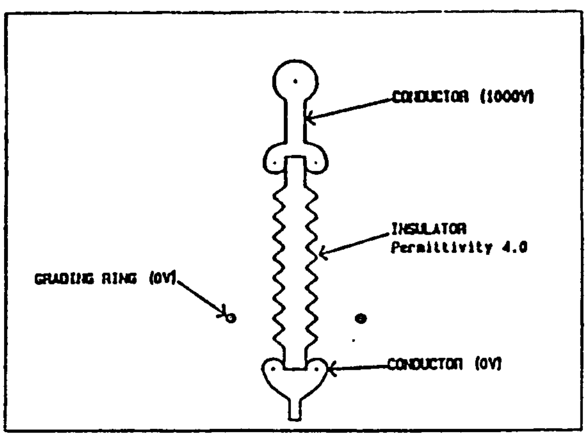

Fig. 3. Sample Problem Geometry

devices stated above. The saw-tooth meridional profile was modeled with splines. The total amount of time to enter the data for this problem was approximately 30 minutes. Each boundary is then constrained. The top conductor is constrained to 1000 volts, the bottom conductor and the grading rings were constrained to 0 volts. The insulator is set to a dielectric constant of 4.0 and a conductivity of 0.3789E-06. Boundary elements are then added to the problem geometry (Figure 4). In this example 129 elements were used. When using the BEM it is important to

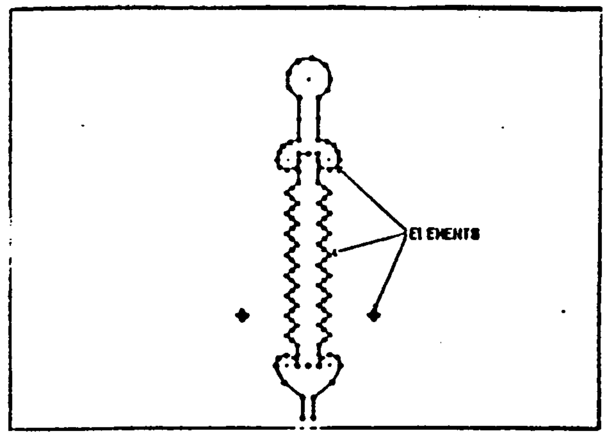

Fig. 4. Sample Problem Elements

"pack" the elements into all corners and triple points. Failure to do so introduces error in the boundary conditions. To solve any problem, the symmetry of the problem must be considered. In this example a two-dimensional type of solver was used, with the result shown in Figure 5. This

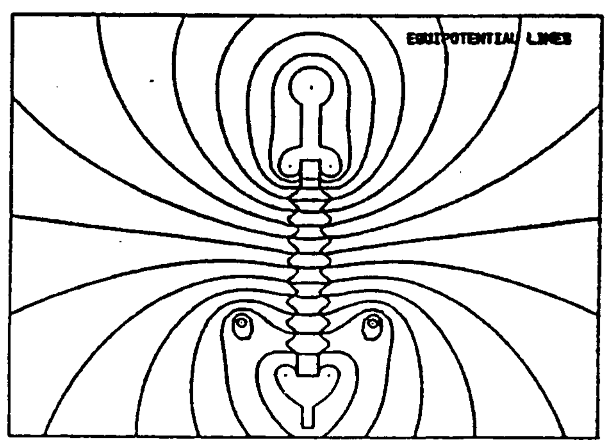

Fig. 5. Sample Problem Solution

type of problem can also be solved axisymmetrically. To do so, the geometry must be rotated about the axis of symmetry and an axisymmetric solver must be used.

B. Example Solved Problems:

Figure 6 shows two conducting rods in a vacuum. This problem was solved

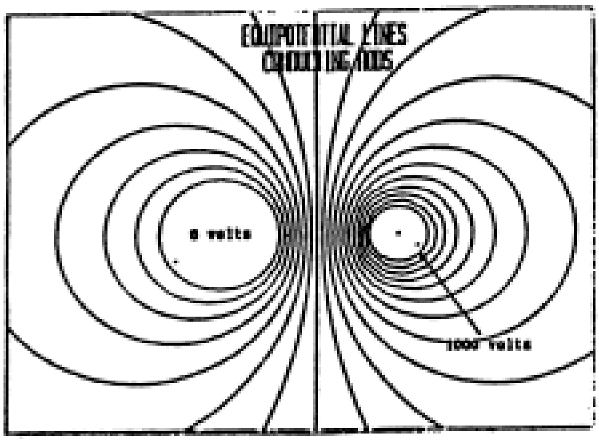

Fig. 6. Conducting Rods

solved in two dimensions. The left rod is constrained to 0 volts and the right rod is constrained to 1000 volts. The solution has seventeen voltage contours each representing a difference of 58.8 volts. The center contour is at 387 volts. This is easily checked within the program by setting the cursor at the desired location and clicking the mouse. All pertinent information is displayed. Figure 7 shows two spheres

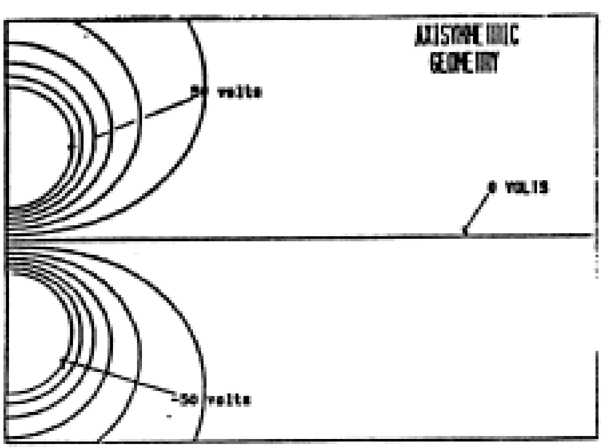

Fig. 7. Axisymmetric Geometry

rotated about a center line. This problem is solved axisymmetrically. The top sphere is constrained to +50 volts and the bottom sphere at -50 volts. The center contour is at 0 volts. In figure 8 the same geometry is used but with the addition of E-Fields represented by arrows. The

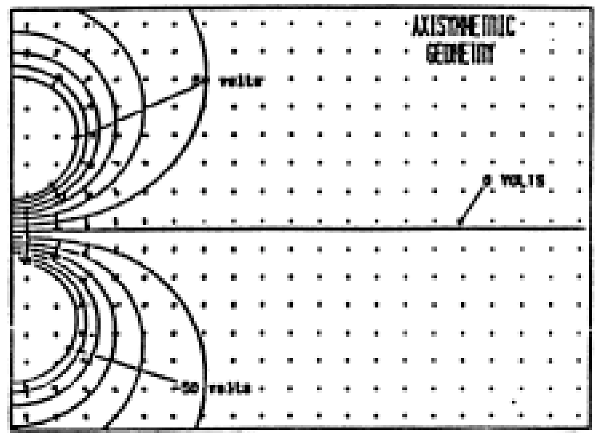

Fig. 8. Contours with E-Field Arrows

length of the arrows indicate the magnitude of the E-Field at the center of the tail. The magnitude of the field at any point can be taken directly from the solution using the same technique as above. Figure 9 uses the same

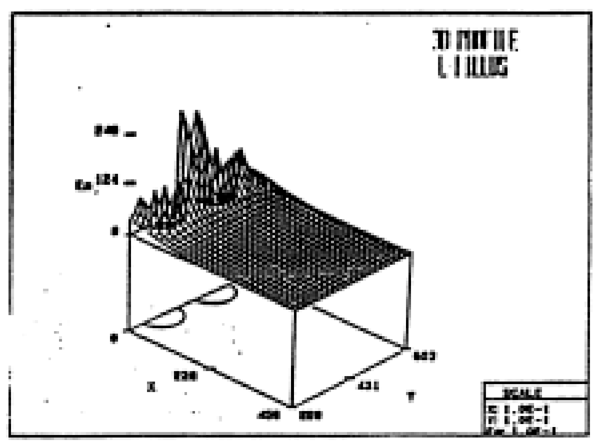

Fig. 9. 3D-Profile of E-Fields

geometry as in Figure 8, but solves for the E-Field magnitude, and is represented in profile form. Figure 10 is one quarter of a pressurized high voltage switch used at the Horizontally Polarized Dipole (HPD) Electromagnetic Pulse simulator (EMP) at Kirtland Air Force Base . The entire switch was entered about it's center axis. This allows half of the geometry to be rotated and solved with the

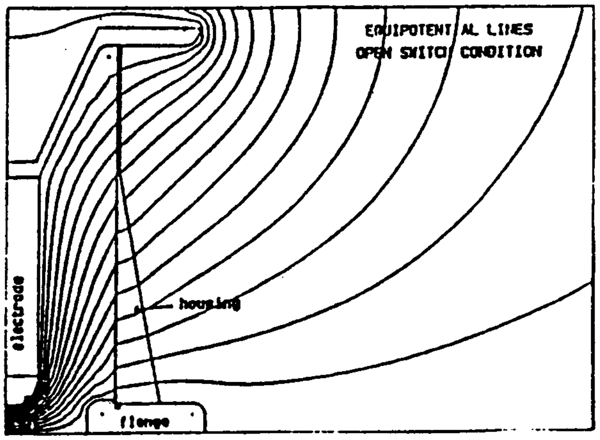

Fig. 10. Open Switch Condition

axisymmetric solver. In this case only one fourth is displayed on the screen, but the entire geometry is known to the solver. The electrodes are constrained to +1000 volts for the top or visible electrode and to -1000 volts for the bottom electrode. The flange is set to 0 volts and is held in place by the housing which is a insulator with a dielectric constant of 4.0 and a conductivity of 0.3789E-06. This problem was solved for equipotential contours surrounding the open switch before closure. This problem shows that even moderately complex problems can be solved quickly and easily using PC based electrostatic codes.

VI. CONCLUSION

Modern microcomputers are capable of performing moderately complex electrostatic analyses using the BEM technique. FEM type electrostatic codes were also evaluated and found suitable for the microcomputer format, but require more time and hardware to solve problems of any complexity. The BEM will solve most problems in a reasonable amount of time with minimum hardware requirements. The user friendly interfaces to PC based codes allows solutions of high voltage problems to be entered from the conceptual stage to the final analyzed model with a minimum amount of data entry.

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