Electric Field Calculations with the Boundary Element Method

ABSTRACT

The boundary element method is used to calculate the electric field profiles at needle tips commonly used for electrical treeing tests. Field distributions are also obtained for polyethylene containing a space charge, at the needle tip, and are compared with the values previously obtained by the finite difference method.

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Electric Field Calculations with the Boundary Element Method

S. S. Bamji, A. T. Bulinski

National Research Council of Canada, Ottawa, Ontario, Canada

and K. M. Prasad

Integrated Engineering Software, Winnipeg, Manitoba, Canada

ABSTRACT

The boundary element method is used to calculate the electric field profiles at needle tips commonly used for electrical treeing tests. Field distributions are also obtained for polyethylene containing a space charge, at the needle tip, and are compared with the values previously obtained by the finite difference method.

THE life of an electrical power device, such as a HV transformer, cable or capacitor, usually depends on the operating state of its electrical insulation. The use of polymeric insulating materials in power cables and capacitors has been widespread. Although these materials have excellent short-term mechanical and dielectric properties, they are susceptible to long-term degradation produced by the action of electric stress. Under normal operating conditions impurities, contaminants or defects accidentally introduced into the polymeric insulation during the synthesis or manufacturing process, can act as points of electric stress enhancement at which the degradation can initiate.

In order to simulate such points of electrical stress enhancement, in most experimental studies, sharp needles are inserted into the polymeric insulation to form needle-plane or needle-needle geometries. The electric fields at such points are usually estimated by equations which do not take into account the permittivity of the insulating material or the space charge effect. It is extremely important to know the values of the local electric fields in the polymeric insulation because they could exceed a certain threshold level above which deterioration processes can initiate [1]. This would ultimately cause the failure of the HV device.

In the past, several methods such as finite difference, finite element, boundary element, boundary integration and charge simulation have been used for solving electric field distributions [2]. The finite difference and finite element methods seek a direct approach for solving the governing differential equation of the potential.

In the finite difference method, the differential operator is separated by utilizing a truncated Taylor series expansion in each coordinate direction and then applied at each point of a rectilinear grid placed on the region under consideration. The method usually involves an iterative process and the main disadvantages are the crude modeling of the sample geometry and large number of unknowns, especially for open-field problems.

The finite element method uses a variational technique in which the potential is approximated by a sequence of functions defined over the entire domain of the defined geometry. By minimizing a function that is proportional to the energy of the system, these approximations are related to an operator equation which yields the values of the solution at the nodes.

In many practical applications, calculation of the potential by interpolation and of its derivatives through differentiation, the derivative has discontinuities in the geometrical model. Moreover, it is not possible to model infinitely extending regions. These problems constitute the major shortcomings of the finite element method.

Both methods have been researched extensively [3] but require large amounts of input data and expensive computer hardware. An alternative approach to the solution of the boundary value problem is the boundary element method which is based on a formulation of the boundary integral equation [4].

This method does not seek a direct solution of the potential. Instead, an equivalent source which would sustain the field is found by forcing it to satisfy a prescribed set of boundary conditions, under a so-called Green's function. This function relates the location and effect of the source to any point on the boundary and eliminates the need for a finite element mesh or a finite difference grid.

The equivalent source usually is located on the boundaries and interfaces of the different media, and once the source is determined the potential or its derivatives can be calculated at any point. Parameters, such as capacitance and inductance, can be obtained.

The main advantages of the boundary element method over the direct approach are the elimination of differentiation and interpolation to calculate the potential or its derivatives and a more accurate modeling of the particular geometry [5].

In the present work electric field profiles are calculated by the boundary element method for needle tips embedded in polyethylene. Field distributions are obtained also for polyethylene containing a space charge at the needle tip, and are compared with the values previously obtained by the finite difference method.

The dimensions of the needle-plane geometry used in this work are shown in Figure 1. A needle was held in air or embedded in a slab of polyethylene ($\epsilon = 2.3$) such that the needle tip was 10 mm from a circular metal plate having a radius of 50 mm. The plate and needle were at ground potential and 10 kV, respectively. The electric field was calculated with the boundary element method by using a commercially available software package 'Coulomb' on a PC 486, operating at 66 MHz, with a 8 MB RAM. A typical calculation for the electric field profile at the tip required ~ 1 h, but the time increased to 30 h for calculations which involved space charge distribution in the solid dielectric.

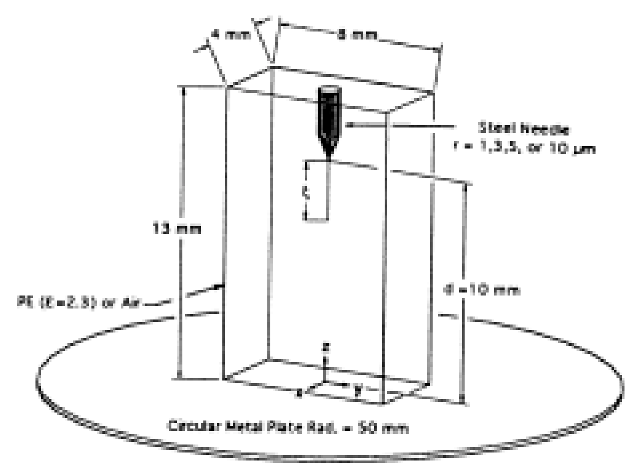

Figure 1.
Dimensions of the needle-plane geometry.

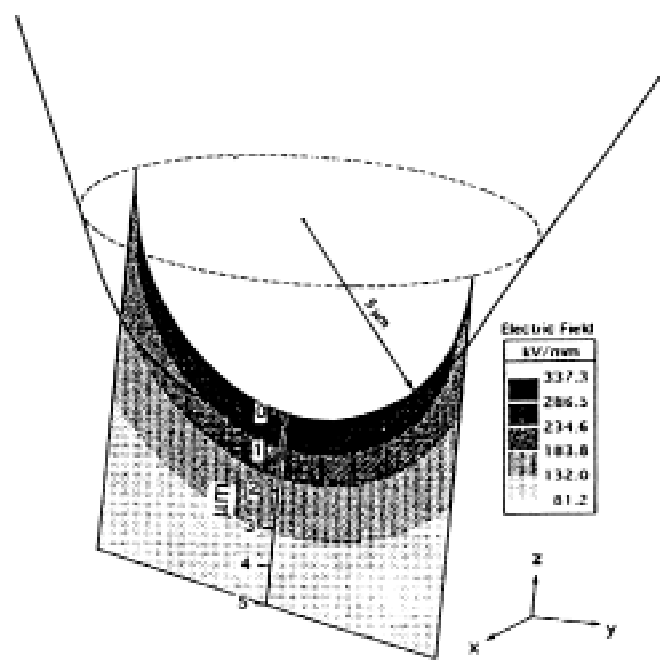

Figure 2.
Electric field profile for a 5 μm needle tip embedded in polyethylene.

The contour plot of the calculated electric field for a 5 μm needle tip embedded in polyethylene is shown in

Figure 2, for a plane through the z-axis. The different shades of grey indicate the magnitude of the highly divergent electric field. Similar results were obtained for the needle held in air except, as expected, the electric field values were larger than in the dielectric. The equipotential line and voltage at any point could also be obtained.

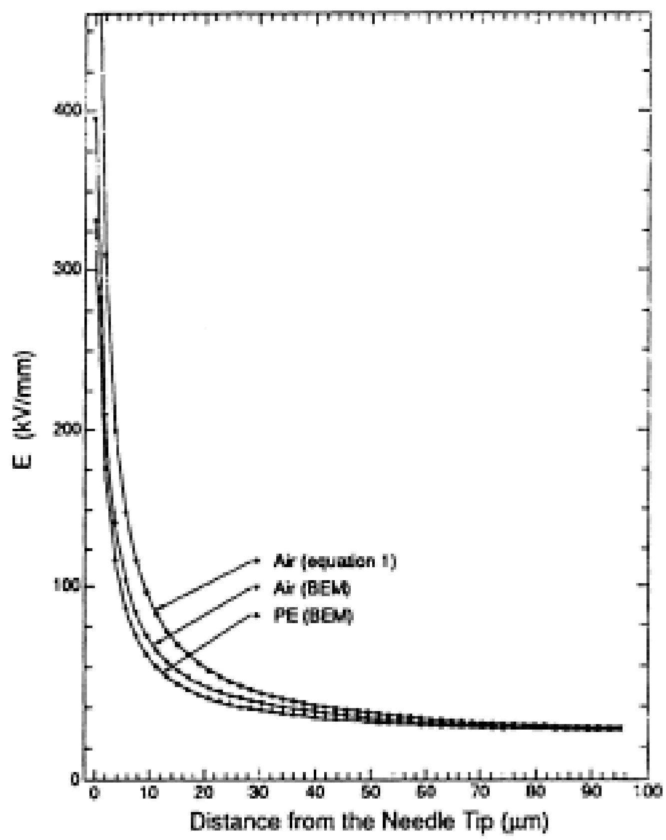

Figure 3.

Electric field as a function of the distance from the needle tip.

The electric field values, for a 5 μm needle held in air and in polyethylene, as a function of the distance along the z-axis are shown in Figure 3. The components (E_x, E_y, E_z) of the electric field could also be obtained for any point. The dotted curve in this Figure is the estimated value of the electric field obtained from the equation

$$E(\xi) = \frac{2V}{[r + 2\xi - \xi^2/d] \ln[1 + 4d/r]} \qquad (1)$$

where $V = 10$ kV is the voltage applied to a needle represented by a hyperboloid projecting towards the plane electrode, $r = 5$ μm is the radius of the needle tip, $d = 10$ mm is the distance of the tip from the bottom surface and ξ is the distance from the needle tip along the z-axis.

Equation (1) was derived from an equation given by Mason [6] for a conducting hyperboloid held at a distance d from a grounded plane electrode. At the needle tip $\xi = 0$, so Equation (1) becomes

$$E_{max} = \frac{2V}{r \ln[1 + 4d/r]} \qquad (2)$$

Equation (2) is commonly used in the literature for estimating the electric field at a needle tip [7–9].

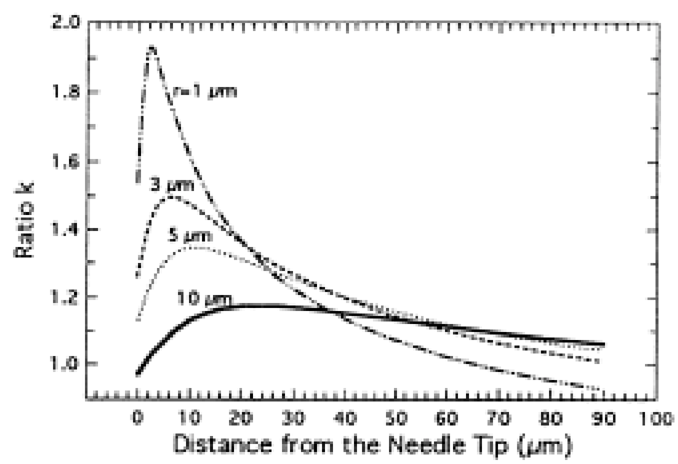

Figure 4.

Ratio k of the electric field values obtained from Equation (1) to those calculated by the boundary element method.

The graphs of Figure 3 show that, for distances < 80 μm from the needle tip, the values of the electric field in air obtained from Equation (1) are larger than those calculated by the boundary element method. The ratio k of the electric field values obtained from Equation (1) to those calculated by the boundary element method is given in Figure 4. The graphs show that as the radius of the needle tip decreases, the ratio increases *i.e.* the difference between the field values estimated by Equation (1) and calculated by the boundary element method becomes larger.

For electrical treeing tests, ac voltage is usually applied to a needle tip embedded in the dielectric. Positive and negative charges can be injected into the material during the positive and negative half cycles of the ac voltage [1]. Some of these charges will be trapped in the various trapping levels of the dielectric to form a space charge at the needle tip. Because of the space charge effect, the actual value of the local electric stress at the needle could be higher or lower than the estimated value.

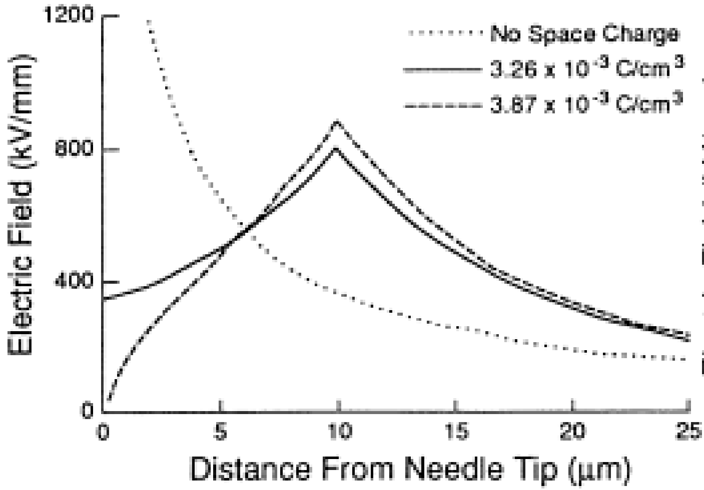

Figure 5.

Electric field as a function of the distance from the needle tip, obtained from the finite element method [10].

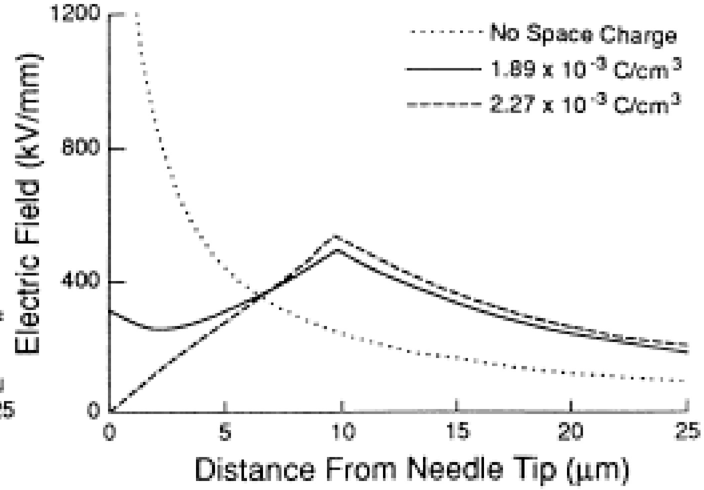

Figure 6.

Electric field as a function of the distance from the needle tip, calculated with the boundary element method.

The finite difference method was used in the past to calculate electric field profiles at a 3 μm needle tip held at 40 kV and embedded in a slab of polyethylene containing a space charge [10]. The needle tip was held 10 mm from the bottom surface of the polymer which was at ground potential. To simplify the calculation, the space charge was assumed to be confined to a cylindrical region around the rotational axis of the needle and was supposed to be uniformly distributed in a cylindrical volume having a radius of 10 μm and a length of 10 μm. The electric field profiles along the z-axis obtained by the finite element method [10], for two different space charge distributions are shown in Figure 5.

The software for the boundary element method, used in the present study, did not have an option for a volume charge density in the dielectric. However, it could accept a surface charge density; hence, the cylindrical space charge region, 10 μm in length, was assumed to be made of very thin disks stacked one over the other and the relevant surface charge was applied to each disk. The electric field profiles along the z-axis calculated by this method for two different space charge distributions are shown in Figure 6.

The graphs in Figures 5 and 6 show that the electric field at the needle tip decreases as the space charge density increases and has a maximum value at the edge of the space charge region. For zero space charge, the values of the electric field previously obtained by the finite element method were larger than those calculated by the boundary element method.

Also, according to the finite element method (Figure 5) a space charge density of 3.87×10^{-3} C/cm^3 was required to reduce the electric field at the needle tip to zero. However, according to the boundary element method (Figure 6), a space charge density of 2.27×10^{-3} C/cm^3 was sufficient to produce the same effect.

In summary, the boundary element method was used to calculate the electric field profiles at needle tips commonly used for electrical treeing tests. It is shown that the boundary element method can account for the presence of the dielectric while Mason's equation [Equation (1)] cannot. The difference between the field values obtained from the boundary element method and Mason's equation becomes significant for tip radii < 10 μm.

Space charge can enhance or reduce the electric field close to the needle tip depending on the nature of the applied voltage. However, the electric field profiles obtained by various methods are highly dependent on the element structure used. Hence, different software packages could yield different results.

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