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INTRODUCTION

The Boundary Element Method (BEM) is a well-established numerical method for simulation of physical phenomena including structural, fluid flow and heat transfer analysis. It is particularly interesting in electromagnetics because of its handling of open region problems and its accuracy in computing the fields where gradients are high. Classical BEM, however, does not inherently contain a facility for dealing with regions with inhomogeneous material properties such as those exhibited by realistic magnetic materials. Furthermore, the concept of a reliable and meaningful error calculation is not as well established as it is with the more common Finite Element Method (FEM). This paper outlines a method for handling the effects of material nonlinearity in magnetics problems. A meaningful way of calculating the error in discrete parts of the model is also presented. Finally, the approach is verified through the comparison of results from a commercial BEM package to an analytic case.

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Simulation of Nonlinear Magnetic Devices Using Boundary Element Method Based Software Tools

T. N. Judge, Enginia Research Inc., Canada

Introduction

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Boundary Element Method

The Boundary Element Method (BEM) is a numerical method used in the solution of boundary value problems. In static magnetics we define a magnetic vector potential, \vec{A} , as:

$$\vec{B} = \nabla \times \vec{A}$$

where \vec{B} is the magnetic flux density. The magnetic vector potential due to an arbitrarily oriented current is calculated by [1]:

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int_S \vec{K}(r') G(r | r') ds + \frac{\mu_0}{4\pi} \int_V \vec{J}(r') G(r | r') dv'$$

where \vec{K} is the real and equivalent current density on the boundaries, S , \vec{J} is the real and equivalent current density through the volumes, V , and G is the free space Green's function:

$$G(r | r') = \frac{1}{2\pi} \ln \frac{k}{|r - r'|}$$

for two-dimensional formulations, and:

$$G(r | r') = \frac{1}{4\pi} \frac{1}{|r - r'|}$$

in three-dimensions.

Finally, \vec{B} will be calculated by:

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int_S \nabla \times (\vec{K}(r') G(r | r')) ds + \frac{\mu_0}{4\pi} \int_V \nabla \times (\vec{J}(r') G(r | r')) dv'$$

The continuity of the normal B-field is satisfied everywhere using this equation, but the tangential H-field must be satisfied as part of the numerical method. That is:

$$J_s = n \times (H_2 - H_1)$$

In the BEM scheme, boundaries where the magnetic properties of materials change are divided into boundary elements, and the regions containing volume sources such as currents are subdivided into smaller regions called sub-areas (figure 1). The elements separating regions with different material properties are then assigned equivalent currents such that the above boundary condition is satisfied given the field equation for \vec{B} .

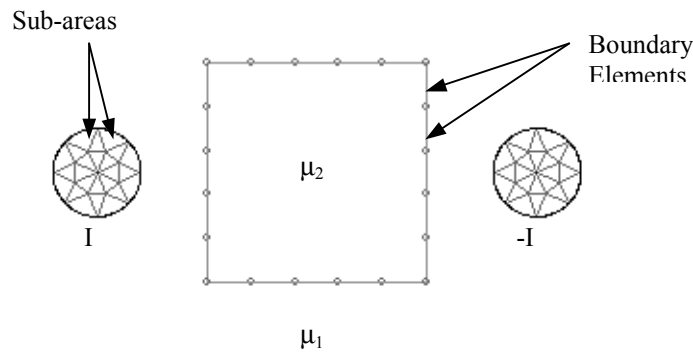


Figure 1: Division of interfaces into boundary elements and regions with real current into sub-areas

Nonlinear Materials

The reality in magnetics problems is that materials used are highly nonlinear in nature. Historically, this has proven difficult for BEM solvers. Because material effects are normally accounted for at the boundary, the nonlinearity and hence the non-homogeneity inside of regions containing magnetic materials cannot fully be calculated by elements on material interfaces alone. This is especially true when the field at various points inside the region forces the material to operate at various levels of magnetic saturation.

One method that has proven very useful for calculating effects of internal nonlinearity is the application of equivalent currents throughout the volume of the nonlinear regions [2]. The fundamental equation to be satisfied in the process is:

$$\nabla \times \vec{H} = J_v$$

Using this concept, the region containing the nonlinear magnetic material is then subdivided in the same manner as the regions containing real volume sources (figure 2). On each of these sub-areas, an equivalent volume current is applied such that over each sub-area:

$$\int_{V_i} \nabla \times \vec{H} dv = \int_{V_i} J_v dv$$

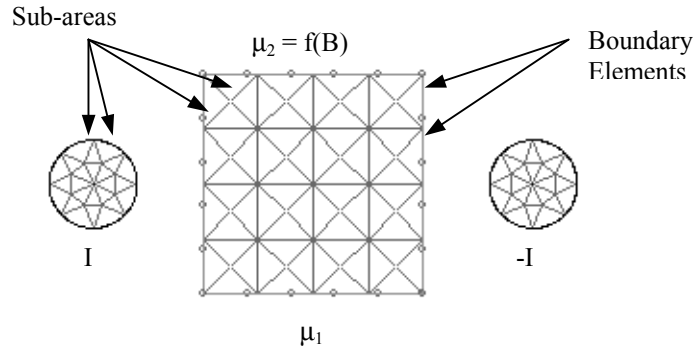


Figure 2: Division of nonlinear regions into sub-areas

The solution of nonlinear problems consists of an iterative process to determine the proper value of the permeability to use at each element on the boundary and to determine what value of equivalent volume current should be used at discrete points in the region of nonlinearity. The iterative scheme used here consists of the following steps:

Use the previous field solution to calculate the permeability at each element – in the first step, there is zero field and thus the initial permeability of the material curve is used.

Use the previous field solution to determine the equivalent volume current in each sub-area.

Use the new values of the permeability at each boundary element and the equivalent volume current to find a new field solution.

Repeat the previous steps until the change in the total solution is less than some prescribed value from one iteration to the next. That is:

$$\sum_{nElements} (K_i - K_{i-1}) + \sum_{nSub-areas} (J_i - J_{i-1}) < \varepsilon$$

where i represents the current step in the iterative process and ε is the user defined exit criterion.

Another important consideration in the practical solution of nonlinear problems is the updating of material information at the boundaries. Each step of the process uses the previous solution to calculate the fields and then locate the operating point on the B-H curve for each discrete element. As the process can, in some steps, calculate unrealistically high values of the field, a method must be used which brings the calculated field values into a more realistic regime. A good candidate for this process is described by Rikabi [3]. At each step, the field is damped to represent a more logical operating point on the curve using the diagram in figure 3. The previous value of the permeability is used to calculate the effective value of the H-field, and then the damped value of B is found by locating the B value on the curve at the new H value.

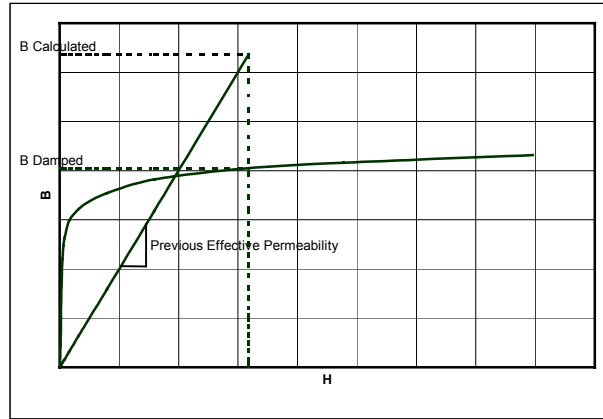


Figure 3: Calculation of the damped B-field from the previous value of the effective permeability and the newly calculated B-field

Unfortunately, this process can be non-convergent when the desired exit criterion is very small. In these cases, however, the damping scheme may be terminated at some point and a more traditional method for updating the boundary permeability can be adopted. That is:

$$\mu_i^j = \mu_{i-1}^j + \alpha(f(|B_i^j|) - \mu_{i-1}^j)$$

where $f(|B_i^j|)$ is the permeability on the j^{th} element during the i^{th} iteration as calculated from the B-H curve given the B-field magnitude, $|B_i^j|$, and α is a relaxation factor, usually $0.5 < \alpha < 1.0$.

Error Calculation and Element Self-Adaptation

Most designers of magnetic devices have no interest in becoming experts in analysis methods such as BEM and FEM. For this reason, it is important for an analysis package to provide meaningful insight to the user as to where the model can be improved. It is especially useful if the package can pick out the largest error, refine the element discretisation and re-solve the model, repeating this until an acceptable level of error has been achieved. This second concept is often referred to as self-adaptation. Two bases for a good self-adaptation algorithm are the ability to calculate the error on discrete parts of the model in a meaningful way, and a parameter to use in deciding when the model has been refined enough to call the error level acceptable.

When the above-described method of solving nonlinear models is used, the error calculation must place a similar importance on the calculation of boundary elements as on the regional elements (called sub-areas). As the solution for both the boundary elements and sub-areas are the equivalent currents, a context in which the error is related to the equivalent current either on the boundary or inside of the nonlinear material would be suitable. Both of these types of error can be calculated from the adherence to fundamental boundary conditions or field equations [4].

The BEM is an error minimization method in the integral sense. That is to say, the continuity of tangential H is satisfied over each boundary element:

$$\int_S J_s ds = \int_S n \times (H_2 - H_1) ds$$

However, this is not to say that everywhere on the element H tangential continuity is satisfied. That is:

$$J_s \neq n \times (H_2 - H_1)$$

This fact can be used to define an error density term over each element:

$$J_E = n \times (H_2 - H_1) - J_S$$

Finally:

$$E_{S_i} = \int_{S_i} |n \times (H_2 - H_1) - J_S| ds$$

expresses the total error on the element, i , as a total current in units of Amperes.

Similarly, the numerical method used to compute the equivalent volume currents due to material nonlinearity enforces curl of H in an integral sense. That is, on each sub-area:

$$\int_V J_V dv = \int_V \nabla \times H dv$$

but:

$$J_V \neq \nabla \times H$$

This allows the definition of the error over each sub-area:

$$E_{V_i} = \int_{V_i} |\nabla \times H - J_V| dv$$

which is once again the error expressed as a total current.

The choice of a parameter for the decision to end the refinement process could be based on the solution from one refinement to the next. For example, a user may wish to know that the calculated magnetic force on a piece of the model does not change by some specific factor from one refinement to the next. The current method compares the total element and sub-area solution to the total element and sub-area error to obtain an error measure:

$$\epsilon = \frac{\sum_{nElements} E_{S_i} + \sum_{nSub-areas} E_{V_j}}{\sum_{nElements} K_{S_i} + \sum_{nSub-areas} J_{V_j}}$$

The process is terminated when the value of ϵ reaches some prescribed value.

Comparison to Analytic Models

A simple analytic model is used to confirm the accuracy of the method for solving models with nonlinear materials and to confirm the validity of error calculation and self-adaptive scheme. Consider a coaxial system as shown in figure 4. The center region is a conductor carrying current, I . The outer shell is a nonlinear material with B-H curve as shown in figure 2. Due to the symmetry of the problem, the magnetic field is easily computed as a function of the distance from the center of the conductor as:

$$H_\phi = I / 2\pi r$$

This is a non-trivial problem for two reasons. Firstly, if the material properties change significantly through the region, their effect cannot be fully accounted for at the boundaries, as previously noted. Secondly, since there is no air gap in the problem, all of the reluctance in the magnetic circuit occurs in the region containing the nonlinear material and therefore, the proper modeling of the nonlinear curve is essential.

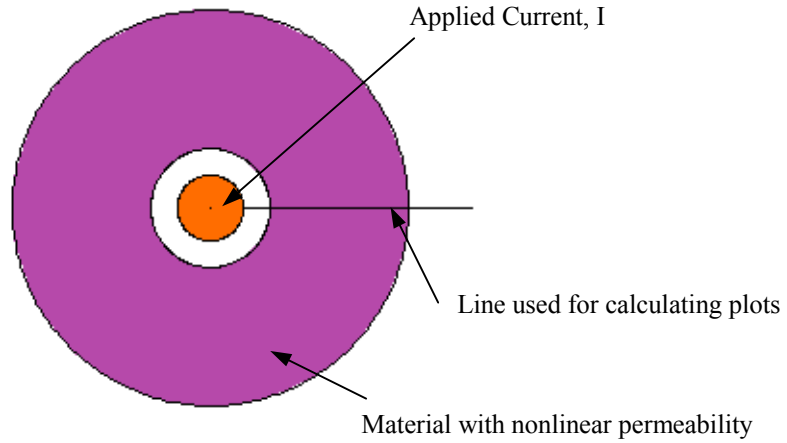


Figure 4: Geometry of the analytic model used

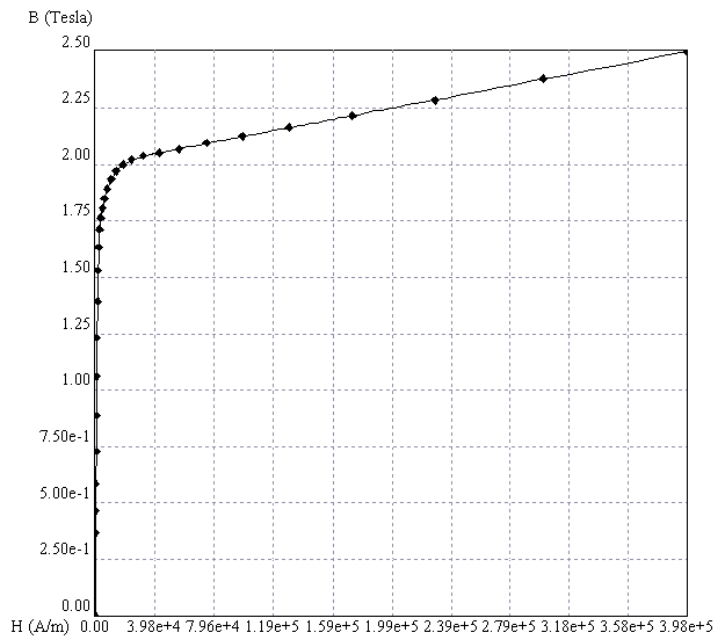


Figure 5: B-H curve for the nonlinear material

At low current levels, the material is operating in the linear regime of the curve and therefore discretisation of the region is not necessary. Similarly, when the current level is extremely high and the material is completely magnetically saturated, the effective permeability is approximately unity and there is no need for discretising the region. Figure 6 shows the analytic and calculated H-field as a function of distance, r , along with the relative absolute error in the calculation at each point. Both of these cases are easily solved with a minimal number of boundary elements and the solution time is literally a few seconds.

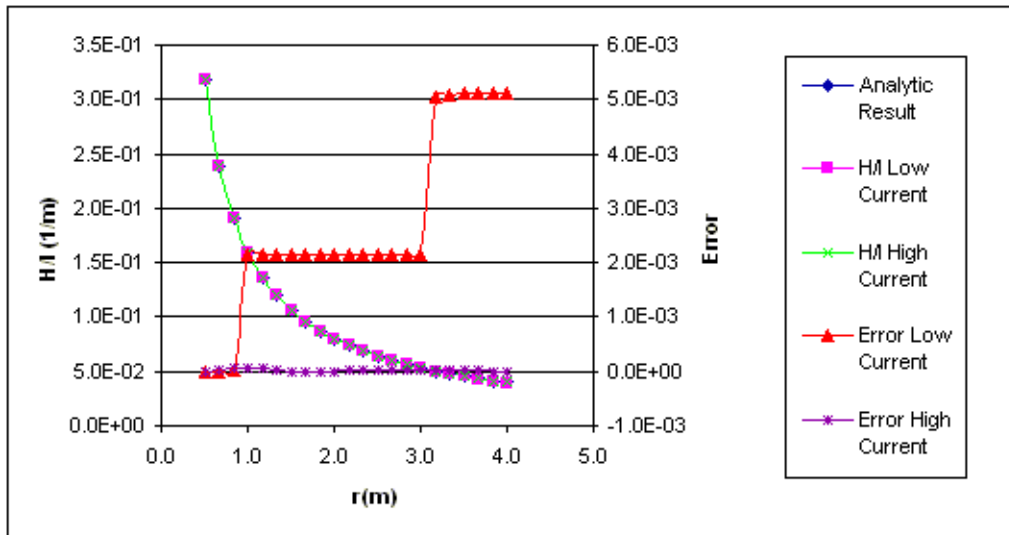


Figure 6: Comparison with analytic results for extremely low and extremely high current

When the material is partially saturated, however, different parts of the model are operating in different regimes of the curve and therefore the requirement for discretisation of the interior of the region begins. Figure 7 compares the analytic result above with a model using three different region discretisation schemes, 0, 232, and 1008 sub-areas in the nonlinear region. For each model the same boundary element division scheme is used and is the same as that used for the high and low current cases.

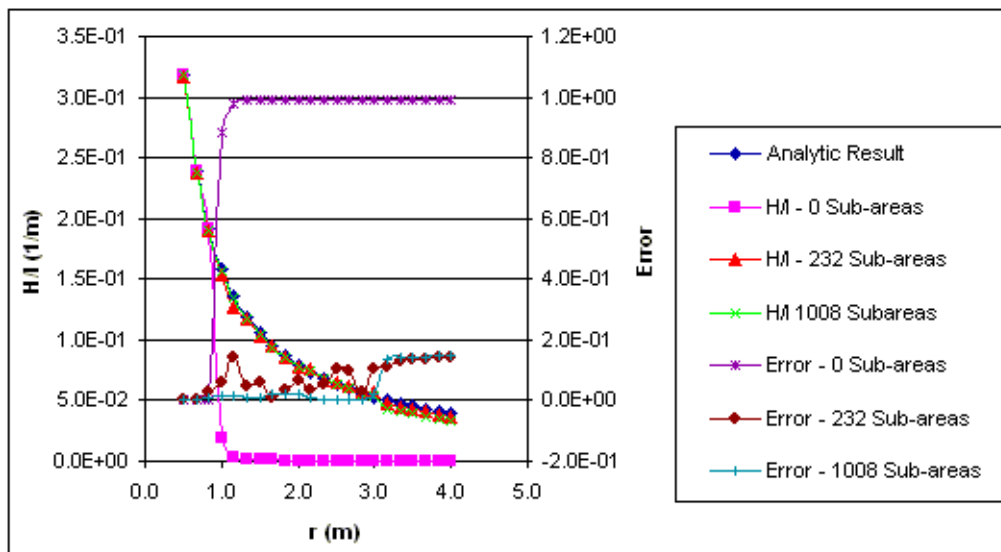


Figure 7: Comparison of analytic result to computed result with varying number of sub-areas

Of course, the denser the region discretisation, the more expensive the calculation becomes and therefore, the need for a non-uniform mesh that is concentrated only on areas where it is required becomes important. Furthermore, because the sub-areas dominate the solution inside the region, but have reduced effect on the field calculated outside of the nonlinear region, the boundary elements must also be refined. Notice that even for the 1008 sub-area case, error is still relatively high near the end of the plot, which is just outside of the region. To this end, a self-adaptation scheme will refine sub-areas only where needed and additionally add boundary elements to the boundaries where a refinement is required.

The error calculation and self-adaptation scheme described above is then used for the same model with varying exit criteria. Results from this series are shown in figure 8.

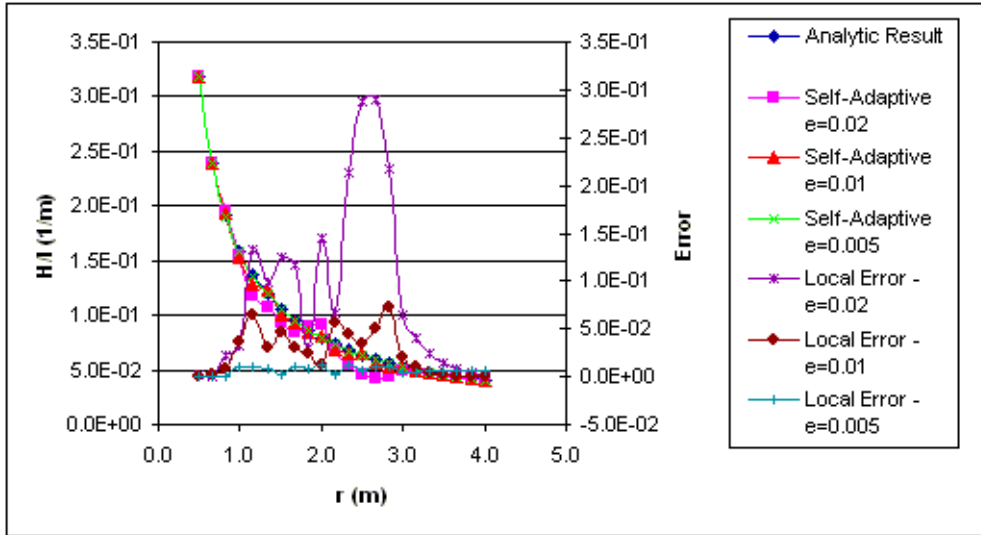


Figure 8: Comparison of analytic result to computed result with varying self-adaptive exit criteria

Total solution times, as well as total number of elements and sub-areas, are given for each exit criteria in table 1.

Exit Criteria	# of Elements	# of Sub-areas	Solution Time (seconds)
0.02	45	76	15
0.01	109	293	55
0.005	278	1276	330

Table 1: Number of elements, number of sub-areas and solution time for each exit criteria

Conclusions

A method for resolving volumetric effects of nonlinear materials using the boundary element method is outlined. The nonlinear method is strengthened by a meaningful way of calculating the error in the model, deciding where to refine the model and deciding when to exit. The method produces near analytic results for problems in which regions containing nonlinear materials are not saturated at all, problems in which the regions are fully magnetically saturated and problems in which there is partial magnetic saturation of a region.

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