Three-Dimensional Electrostatic Field Analysis on the Microcomputer

ABSTRACT

Electromagnetic design parameters can be obtained to a good approximation through two-dimensional modeling of field distributions. Many times the physics of the problem requires three-dimensional analysis. Due to the large amount of data and long modeling and solution times, three-dimensional analysis has been restricted to mainframe, mini and supermicro computers. The application of the boundary element method in an easy-to-use computer-aided field analysis package circumvents the drawbacks of older methods and enables engineers to accurately analyze three-dimensional fields on state-of-the-art microcomputers.

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THREE-DIMENSIONAL ELECTROSTATIC FIELD ANALYSIS ON THE MICROCOMPUTER

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Electromagnetic design parameters can be obtained to a good approximation through two-dimensional modeling of field distributions. Many times the physics of the problem requires three-dimensional analysis. Due to the large amount of data and long modeling and solution times, three-dimensional analysis has been restricted to mainframe, mini and supermicro computers. The application of the boundary element method in an easy-to-use computer-aided field analysis package circumvents the drawbacks of older methods and enables engineers to accurately analyze three-dimensional fields on state-of-the-art microcomputers.

INTRODUCTION

Design and analysis of electromagnetic equipment and components requires accurate calculation of field distributions and device parameters. Engineers use many different methods to this end. Broadly, these methods can be classified as analytical, experimental and numerical methods.

Method of images, conformal mapping, power series expansion, etc. are analytical methods which are used to obtain closed form solutions. Analytical methods have limited uses. Their applicability is severely restricted in problems with complicated geometries. Excessive approximations usually result in arguable results. Nevertheless, they help test numerical methods and form a group of additional tools for engineering design.

Experimental methods are not often cost effective in the initial design stage. Building many different models and testing them one by one is long and arduous. Many times results obtained for a miniaturized model are not necessarily true for the actual device.

Numerical simulation of electromagnetic field distributions has been an area of intense research. Different numerical methods have been developed and implemented in computer-aided design packages to provide engineers with more effective tools. Up to now, they have been available only on mainframe, mini and supermicro systems. With the proliferation of powerful desktop personal computers and new field simulation techniques, computer-aided design is available on microcomputers.

Finite differences, finite elements and boundary elements are three popular numerical methods in use today.

The finite difference method requires the generation of a grid in every region [1]. This results in a large number of unknowns and inaccurate modeling of device boundaries. Fields are calculated by numerically differentiating potential. This gives rise to inaccurate results.

The finite element method requires the division of regions into a mesh [2]. This takes a long time, requires considerable effort from the user and results in a large number of unknowns.

The above disadvantages may account for the fact that three-dimensional field simulation could only be done on large mainframe computers by finite element experts.

BOUNDARY ELEMENT METHOD

The boundary element method solves the integral equation formulation of the field and has the unknowns only on boundaries [5]. This leads to significant ease of use, less input data and extremely good accuracy [3],[6].

The boundary element method produces precise results with far less data as compared to the methods of finite differences and finite elements. The boundary element method caters to open region problems without any artificial truncation of the region and models problem geometries accurately [5],[6].

An equivalent source which would sustain the field is found by forcing it to satisfy prescribed conditions under a free space Green's function which relates the location and effect of the source to any point on the boundary.

The use of Green's function effectively eliminates the need for a finite element mesh or a finite difference grid.

Once the source is determined, potential and field are computed by integrating the source without interpolation. This provides inherent stability.

ELECTROSTATIC FIELD EQUATIONS

For the electrostatic field, we seek the solution of the scalar potential due to some sources such that

$$\underline{E} = - \nabla \Phi \qquad (1)$$

For a point source in free space, the electric field has a component in only the radial direction given by

$$\underline{E} = \frac{q}{4\pi\epsilon r'^2}\underline{a}_r = -\frac{d\phi}{dr'} \qquad (2)$$

Choosing a point $\underline{r}_o$ to be the location of the reference potential and $\underline{r}$ the observation point, and integrating along the line joining the observation and reference points yields

$$\phi(\underline{r})-\phi(\underline{r}_o) = \frac{-q}{4\pi\epsilon}\int_{r_o}^{r}\frac{dr'}{r'^2} = \frac{q}{4\pi\epsilon}\left(\frac{1}{|\underline{r}|}-\frac{1}{|\underline{r}_o|}\right) \qquad (3)$$

As the choice of reference potential is arbitrary, it can be set to zero in which case

$$\phi(\underline{r}) = \frac{q}{4\pi\epsilon}\left(\frac{1}{|\underline{r}|}-\frac{1}{|\underline{r}_o|}\right) \qquad (4)$$

If the medium is linear, isotropic and more than one point source exists, then by superposition

$$\phi(\underline{r}) = \sum_{i=1}^{n}\frac{q_i}{4\pi\epsilon}\left(\frac{1}{|\underline{r}_i|}-\frac{1}{|\underline{r}_o|}\right) \qquad (5)$$

where $\underline{r}_i$ is the vector joining the position vector and the charge q_i.

For a continuous source density distribution, the potential is

$$\phi(\underline{r}) = \frac{1}{4\pi\epsilon}\int_{\partial v'}\sigma(\underline{r}')\left[\frac{1}{|\underline{r}-\underline{r}'|}-\frac{1}{|\underline{r}_o-\underline{r}'|}\right]dv' \qquad (6)$$

If the reference point is chosen to be at infinity, equation (6) then reduces to

$$\phi(\underline{r}) = \frac{1}{4\pi\epsilon}\int_{\partial v'}\frac{1}{|\underline{r}-\underline{r}'|}\sigma(\underline{r}')\,dv' \qquad (7)$$

For ease of notation we assume the permittivity $\epsilon=1/4\pi$ in which case

$$\phi(\underline{r}) = \int_{\partial v'}\frac{1}{|\underline{r}-\underline{r}'|}\sigma(\underline{r}')\,dv \qquad (8)$$

The integrand thus contains the distributed source and the free space Green's function in three-dimensions.

From (8), given the source configuration, the potential can be found everywhere in the region. Usually, however, the source is not known but the potential or its normal derivative are specified on the boundary and we seek an equivalent source that will sustain these conditions.

For Dirichlet (Voltage) boundaries, the equation to be enforced is

$$\int_{\partial b} G(\underline{r},\underline{r}')\sigma(\underline{r}')\,ds' = \phi(\underline{r}') \quad \underline{r}\in\partial b \qquad (9)$$

where $G(\underline{r},\underline{r}')$ is the Green's function, $\sigma(\underline{r}')$ is the equivalent source, and ∂b is taken over all boundaries and interfaces.

For Neumann boundaries (where the normal derivative of the voltage specified), a Fredholm equation of the second kind results [4]

$$\phi_i'(\underline{r}) = \int_{\partial b} G_i'(\underline{r},\underline{r}')\,\sigma(\underline{r}')\,ds' + 2\pi\sigma(\underline{r}) \quad \underline{r}\in\partial b \qquad (10)$$

in which $G_i'(\underline{r},\underline{r}')$ is the normal derivative of the Green's function with respect to the unprimed variable.

Along any interface, the continuity of flux is enforced yielding

$$(\epsilon_1-\epsilon_2)\int_{\partial b} G_i'(\underline{r},\underline{r}')\,\sigma(\underline{r}')\,ds' + (\epsilon_1+\epsilon_2)\,\sigma(\underline{r}) = 0 \qquad (11)$$

where ϵ_1 and ϵ_2 are the permittivity values of the materials forming the interface.

The integral equations are discretized along boundaries and interfaces using the Galerkin method. The Galerkin method is one of the projection methods which are also called method of weighted residuals or moment methods [7].

Boundaries and interfaces are divided into small sections which are referred to as boundary elements. This discretization results in a set of linear simultaneous equations for the unknown equivalent charge density coefficients. The solution of this system of equations yields the equivalent source density distribution.

The set of linear simultaneous equations can be solved directly or iteratively. A direct solution takes longer than the iterative solution and may be subject to truncation and round-off errors. An iterative solution is faster than the direct solution for large problems and is not affected by round-off errors.

Once the source is determined, the potential or derivatives of the potential can be calculated at any point. Parameters such as capacitance and inductance are computed directly. Provided the problem is piecewise homogeneous, the equivalent source is located only on the boundaries and interfaces of different media.

Representing the potential at each point by a phasor, steady state sinusoidal fields can be calculated with ease. For example, multiphase transmission line fields can be calculated by solving a set of real and imaginary equivalent sources for given complex boundary conditions.

The boundary element method has been implemented, on a microcomputer, in the program COULOMB. The geometry of the problem that can be solved is arbitrary.

SOLUTION ACCURACY

The accuracy of the solution is the most important criterion for the selection of the numerical technique. Once the problem is solved, how do we know if the results are correct? If the results are not correct, what can be done to improve them? With the boundary element method, one has very powerful options to test the accuracy of results.

Once the equivalent charge is determined, the program may be asked to calculate values on the boundaries. The difference between the assigned and the calculated values on the boundary represents the largest error in the solution.

This is due to the fact that the solutions of Laplace's equation are harmonic functions. Harmonic functions have their maximums on the boundary. Hence, the error which is the difference between the actual and calculated solution is largest on the boundary.

This check is not available with finite element or finite difference methods because the results are interpolated from the nodal values. One, therefore, would get the assigned value exactly.

Field values inside conductor boundaries can be checked to see how close to zero they come.

Along interfaces, the calculated and actual field discontinuities can be checked. The ratio of normal components of the field on either side is inversely proportional to the ratio of respective permittivities.

If the results are not satisfactory, they can be improved by increasing the number of boundary elements on the surfaces and re-solving the problem.

MICROCOMPUTER IMPLEMENTATION

Modern microcomputers are rendered daily more powerful; computations that were once possible only on mainframes and mini-computers can now be handled with ease at a workstation equipped with a personal computer. The new micros offer high speed, accuracy and highly interactive graphics capabilities. The sophistication of the results provided is limited only by the scope of the software used.

An electrostatic field problem is described by defining the geometry, material properties and boundary conditions. It is the geometry of the problem which usually determines if a two- or three-dimensional analysis is most appropriate.

The simulation of fields on the microcomputer requires the input of the above, the numerical solution of the field equation and output of desired parameters. The process is repeated until optimum values for the design parameters are obtained. The efficiency of the procedure is measured by the amount of time required to complete the design.

The factors that affect the efficiency are the ease of use, the accuracy of results, the capabilities and speed of the program.

Industrial users agree that perhaps the most significant issue is the time required to define the problem. In a boundary element package, the user does not mesh every region. This is by far the biggest time saver.

Ease of use is not a concept totally isolated from the numerical technique used. There are a number of important factors which are brought about by the solution technique used in the package which contribute to the ease of use.

The user interface in COULOMB was designed to require minimal keyboard entry and hand motion. Menus are structured to follow the natural pattern of defining and solving a problem and to incorporate the same sets of commands that operate on different objects. On-line help is provided in every menu.

The geometry, material properties, and other information can be entered with the help of a mouse and immediately displayed. This approach minimizes human error and the time required to enter or modify a particular problem.

The special features of the microcomputer environment (e.g. fast color graphics, color printer, mouse or keyboard entry, math co-processor, hard disk, RAM disk) are fully utilized to integrate problem definition, analysis, data storage, drafting and presentation capabilities.

APPLICATIONS

Four sample problems which have been solved using COULOMB are presented below. All calculations were done and all illustrations were generated using COULOMB on IBM PC-type microcomputers.

A High Voltage Test Arrangement

This problem was extracted from reference [8]. The geometry, boundary conditions and an equipotential contour map on a vertical plane passing through the center of the assembly are shown in Fig. 1.

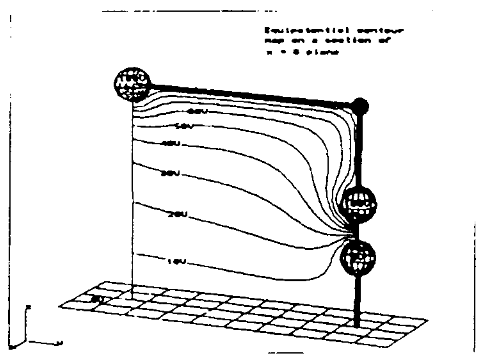

Fig. 1: Geometry, electrode voltages and equipotential contour map.

The geometry of the problem was entered interactively, using a mouse, keyboard and cursor keys. The spheres and cylinders were created with one command. Surface patches (which are modeled as Coons patches) were defined from segments. Simply- or multiply-connected regions were defined from surface patches. Boundary conditions were assigned by selecting the surface patches or volumes using the mouse. Boundary elements were generated only on surfaces between dissimilar media and on surfaces with boundary conditions.

After the program determined the equivalent current distribution, various parameters were calculated. Fig. 2 shows an arrow plot of the electric field on a horizontal plane halfway between the two spheres on the right.

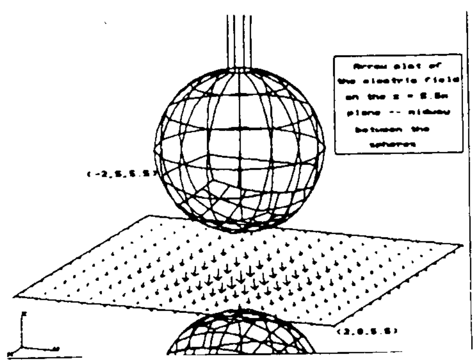

Fig. 2: Arrow plot of the electric field.

With COULOMB, voltage and field values can be calculated at any point. Graphs of field values can be obtained along any arbitrary line or curve on the surface. Arrow plots of the electric field can be calculated on any plane. Fig. 3 shows an equipotential contour map on a vertical plane in the immediate vicinity of the two spheres.

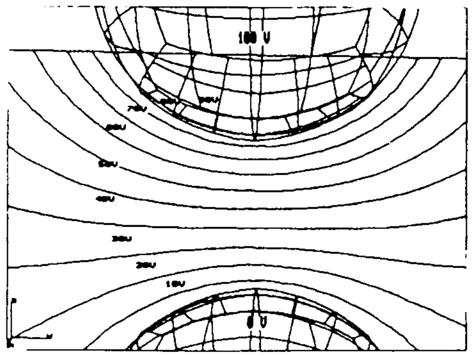

Fig. 3: A close-up of the voltage contours in the vicinity of the spheres.

A Raised Rim Focusing Lens Design

The electrostatic field distribution is calculated for the optimum design of a focusing lens. The geometry of the problem is shown in Fig. 4. Two components are placed further apart than actual for ease of identification. The left component is at 7110V while the right component is at 25000V.

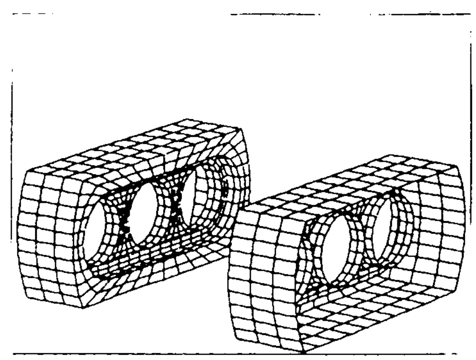

Fig. 4: Raised rim electrode geometry.

Modeling the problem required about two and a half hours. 1168 boundary elements with 1968 unknowns were used for the solution.

Fig. 5 shows the electric field magnitude contours on a plane along the front face of the right component.

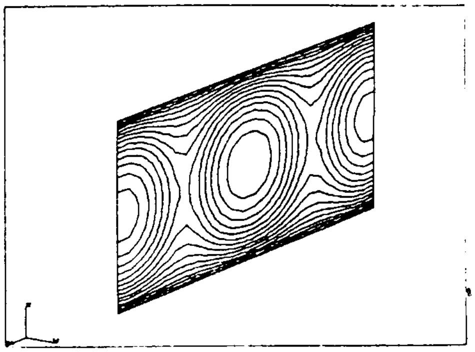

Fig. 5: Electric field magnitude contours.

A HV Power line - Tower Connection

The electrostatic field distribution was analyzed around a high voltage power transmission line which is connected to the tower through a cylindrical insulator with a permittivity of 4.5. The geometry and boundary conditions are shown in Fig. 6.

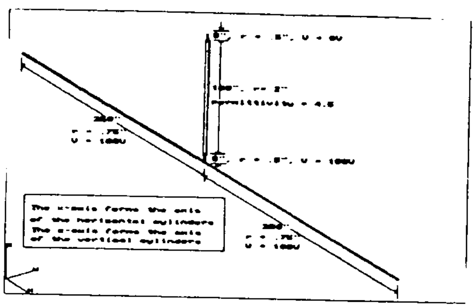

Fig. 6: Geometry and boundary conditions for the power line connected to a tower.

The geometry of the problem consisted of cylinders and planes. Approximately two hours were required to enter the problem.

Fig. 7 shows the voltage variation along the axis of the insulator.

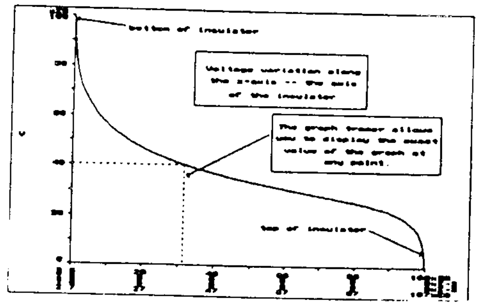

Fig. 7: Graph of voltage along the axis of the insulator.

The equipotential contour lines In close proximity of the conductor-insulator interface are shown in Fig. 8. The vertical contour plane passes through the center of the structure.

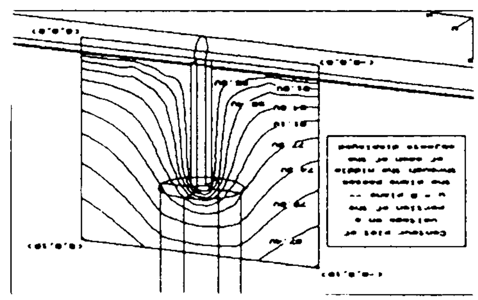

Fig. 8: Equipotential contours around conductor-insulator connection.

The boundary element method calculates both the near and far fields at one time with the same accuracy. One can zoom into any area and obtain accurate results. In the FEM, if one were to zoom into a single element, one would only obtain interpolations from the nodal values of that element.

A Cathode Ray Tube Field Analysis

The electrostatic field distribution was analyzed for a CRT design. High accuracy in field values was required for precise determination of beam trajectories. The CRT geometry and voltages are shown in Fig. 9.

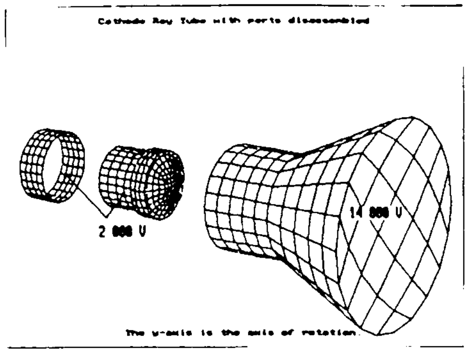

Fig. 9: Geometry and boundary conditions for CRT field analysis.

Engineers are usually required to provide design tolerances. These are difficult to determine and may give rise to quality control problems. The CRT problem in this example was first modeled and analyzed in a rotationally symmetric configuration. Subsequent analyses were performed to obtain answers to a number of what-if questions.

Fig. 10 shows a contour map of the z-component of the electric field on a plane perpendicular to the axis of the CRT.

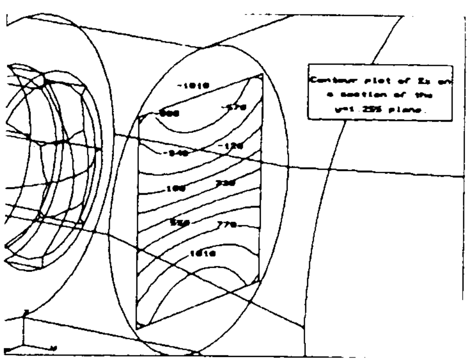

Fig. 10: Contour lines of E_x on a vertical plane.

The boundary element method provides very accurate results as all parameters are calculated by integrating the equivalent charge. In finite elements, fields are calculated by numerically differentiating potential which may give rise to erroneous results.

Fig. 11 shows the graph of the y-component of electric field along the axis of the tube.

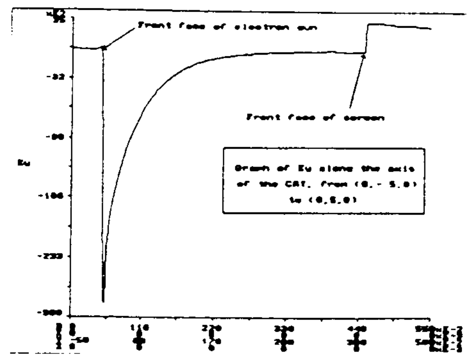

Fig. 11: Graph of E_y along the axis of the CRT.

The problem took about two hours to model, 645 boundary elements with 1072 unknowns were used. The solution time on a 25 MHz 386 PC computer equipped with a math co-processor was 2 (two) hours and 14 minutes.

CONCLUSIONS

The boundary element method has been shown to be an efficient technique for the solution of Laplace's equation in three-dimensional space.

The main advantages of this method are the reduction of one in problem dimensionality, accurate modeling of geometry, elimination of differentiation and interpolation to calculate potential or its derivatives, precise results due to the smoothness of the integral operator and sound means for checking the accuracy of the solution.

Capacitance, inductance, and other parameters are calculated by integrating the free charge which is derived from the equivalent source.

The boundary element method, combined with a highly interactive user interface on a personal computer, automates the analysis of three-dimensional electrostatic field distributions and provides accuracy, speed and ease of use.

Problem geometries, materials and boundary conditions can be conveniently described from the conceptual stage and analyzed to obtain the optimum design.

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REFERENCES

[1] G.E. Forsythe and W.R. Wasow, Finite-Difference Methods for Partial Differential Equations. New York: John Wiley & Sons, 1960.

[2] O.C. Zienkiewicz, The Finite Element Method in Engineering Science. New York: McGraw-Hill, 1971.

[3] Y.B. Yildir, "Computer-Aided Field Analysis of High Voltage Apparatus Using the Boundary Element Method", Proc. International Coil Winding Conference, Rosemont, Illinois, Oct. 5-8, 1987.

[4] S.G. Mikhlin, Integral Equations. New York: Pergamon Press, 1964.

[5] Y.B. Yildir, A Boundary Element Method for the Solution of Laplace's Equation in Three-Dimensional Space. Ph.D. Dissertation, University of Manitoba, 1985.

[6] Y.B. Yildir, "Computer-Aided Magnetic Field Analysis Using the Boundary Element Method", Proc. International Coil Winding Conference, Cincinnati, Ohio, Oct. 5-8, 1988.

[7] R.F. Harrington, Field Computation by Moment Methods. Reprinted by R.F. Harrington, R.D.2, West Lake Road, Cazenovia, N.Y., 1968.

[8] N. Morales, E. Asenjo and J. Inostroza, "A Generalized Analytical-numerical Method for the Calculation of 3-dimensional HV Fields", IEEE Transactions on Electrical Insulation, Vol.23 No.6, December 1988.