USE OF THE BOUNDARY ELEMENT METHOD FOR PULSED POWER ELECTROMAGNETIC FIELD DESIGNS

ABSTRACT

The Boundary Element Method (BEM) is a numerical technique for solving Boundary Integral equations. In this technique electromagnetic phenomena are mathematically described by Maxwell’s equations in integral form. Enforcing the boundary conditions along the material interfaces allows one to obtain a set of boundary integral equations with the unknowns as the equivalent sources or field variables along the interface. One may then separate the boundaries into boundary elements, represent the unknowns on elements, and obtain a system of linear equations. All field variables at any point in space may be obtained by performing integrations associated with the equivalent sources or fields on the boundaries. The BEM is a valuable technique in the electromagnetic field modelling carried out by many engineers during the design phase of their work. The Boundary Element Method has found wide application in fields as diverse as medical, power, defence, research and education engineering design, from the modelling of components (motors, insulators, bushings, lasers etc...) to complete systems, with many pulsed power laboratories and high voltage industries utilising it in various electromagnetic research and development programmes.

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USE OF THE BOUNDARY ELEMENT METHOD FOR PULSED POWER ELECTROMAGNETIC FIELD DESIGNS

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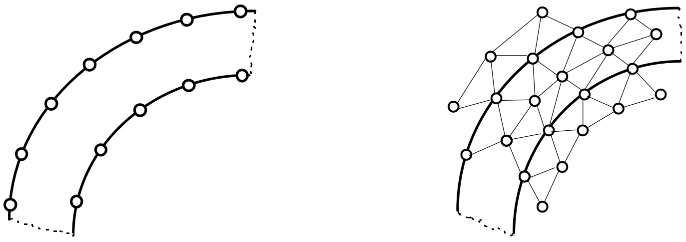
Abstract

The Boundary Element Method (BEM) is a numerical technique for solving Boundary Integral equations. In this technique electromagnetic phenomena are mathematically described by Maxwell's equations in **integral** form. Enforcing the boundary conditions along the material interfaces allows one to obtain a set of boundary integral equations with the unknowns as the equivalent sources or field variables along the interface. One may then separate the boundaries into boundary elements, represent the unknowns on elements, and obtain a system of linear equations. All field variables at any point in space may be obtained by performing integrations associated with the equivalent sources or fields on the boundaries. The BEM is a valuable technique in the electromagnetic field modelling carried out by many engineers during the design phase of their work. The Boundary Element Method has found wide application in fields as diverse as medical, power, defence, research and education engineering design, from the modelling of components (motors, insulators, bushings, lasers etc...) to complete systems, with many pulsed power laboratories and high voltage industries utilising it in various electromagnetic research and development programmes.

As electromagnetic field simulation enters the mainstream of computer-aided engineering, the boundary element method is emerging as an efficient alternative to FEM. This paper describes the boundary element technique in detail and includes a comparison with electromagnetic analysis using the finite element method.

Introduction

The two most widely used methods for solving Maxwell's equations are the boundary element method (BEM) and the finite element method (FEM). Unlike BEM, the FEM is a numerical technique for solving Maxwell's equations in **differential** form. For a given design, the FEM requires the entire design, including the surrounding region, to be modelled with finite elements. A system of linear equations is generated to calculate the potential (scalar or vector) at the nodes of each element. Therefore the basic difference between these two techniques is the fact that BEM only solves the unknowns on the boundaries, whereas FEM solves for the whole space as shown in Fig.1. In pulsed power, surface roughness effects, triple junction effects and surface flashover are the main causes of system failure at elevated stress levels. Analysis techniques such as the BEM which clearly define these surfaces, allows accurate analysis to be made of these potentially weak areas.



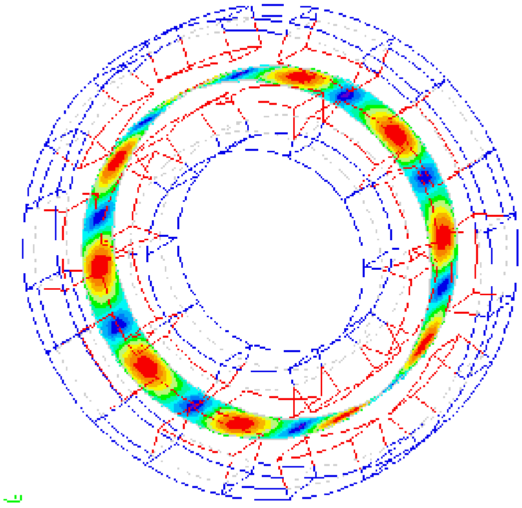
BEM FEM

BEM model versus FEM model

Fig.1

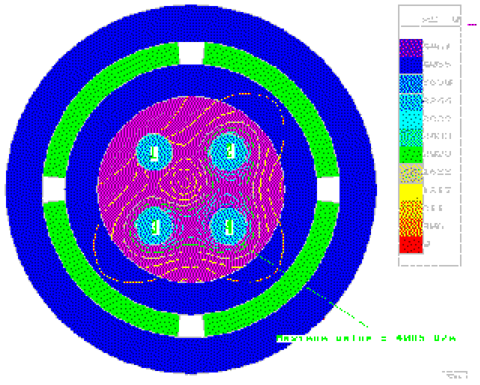
Advantages of the Boundary Element Method

Unlike FEM, which must use a 3D finite element mesh in the whole space, BEM uses only 2D elements on the surfaces which describe the material interfaces or assigned boundary conditions. Therefore users can set up a truly representative system for analysis. Since only elements on interfaces are involved in the solution procedure, system alterations do not require re-meshing. For example, in motor design optimisation, solutions are required for different rotor positions. Using BEM software, only one boundary element distribution is necessary to solve all the rotor positions, and no element reassignments are required. With FEM software, finite elements in the whole space must be re-generated for every new rotor position. Some examples of the various types of analysis which can be carried out are depicted in Figs.2, 3, 4 and 5. This includes electrostatic, magnetostatic, time domain and thermal analysis in both 2 and 3 dimensions.



3D magnetic clutch B-field contour ring

Fig.2

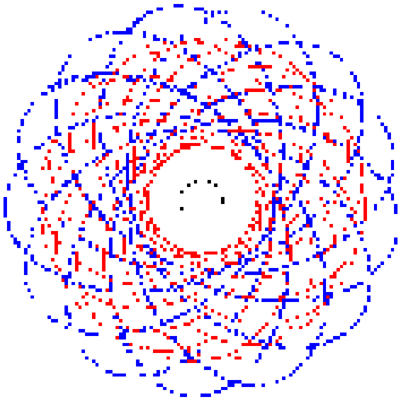


2D cable system E-field colour lines

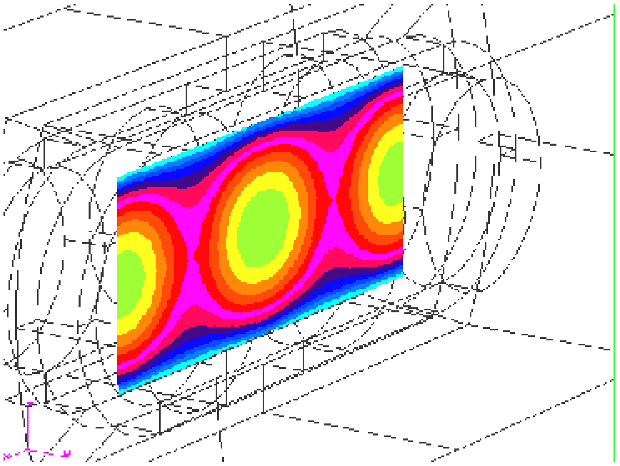
Fig.3

Accuracy of the Boundary Element Method

BEM allows all field variables at any point in space to be calculated accurately. Also, the solutions tend to be precise due to integral operation. Moreover, the unknown variables used in the BEM are the equivalent currents or equivalent charges which have a physical interpretation. By using these physical variables, global quantities such as force, torque, stored energy, inductance and capacitance can be obtained through very simple procedures.



2D trajectory calculations using
Newtonian or Relativistic analysis
Fig.4



3D CRT system
E-field colour centre
Fig.5

Analysis of Open Boundary Problems

The analysis of unbounded structures (e.g. electromagnetic fields exterior to a strip-line or capacitor bank) can be solved by BEM without any additional effort as the exterior field is calculated in the same way, in fact the field at any point in space can be calculated. Therefore for any closed or open boundary problem, users of the BEM need only deal with the real geometric boundaries of the actual pulsed power system. In contrast, open boundary problems tend to be problematic for the FEM since artificial boundaries, which are far away from the real structure, must be introduced. How to determine these artificial boundaries becomes a major difficulty for FEM based software users. Since most electromagnetic field problems are associated with open boundary structures, the BEM is the most appropriate method for general field problems.

Error Analysis Using the Boundary Element Method

From Green’s theorem one can show that if and only if the solution satisfies the boundary conditions on all the boundaries, the result at any point in the solution space obtained from the variables on the boundaries must be correct. Therefore, after solving a problem with a certain element distribution, users can perform an error analysis by checking the boundary values along the actual boundaries. One can improve the solution by simply adding more elements to define the boundary where an error has been found. As the largest errors must occur on the defined boundary, the ability to dictate the level of acceptable error, through element density control, allows the maximum error for the full 2D/3D solution to be controlled.

The Solution of Non-Linear Problems Using the Boundary Element Method

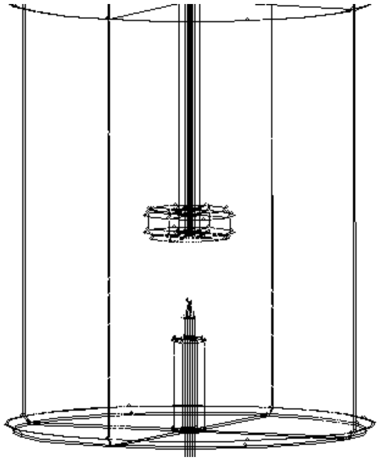
From basic field theory, any magnetised body will produce a magnetic field which can be exactly modelled by a set of equivalent surface and volume currents on and within the structure. This is true whether the materials are linear or non-linear. In cases where it is assumed that the materials are linear, it can easily be shown that the equivalent volume current will be zero and that only the equivalent surface current will be present.

For linear problems, it is precisely these equivalent surface currents which are calculated in magnetostatic problems. A matrix of linear equations is generated from the boundary elements and the matrix is subsequently solved, to determine the equivalent approximation to the surface currents. Any field parameter can then be found via integration.

For non-linear problems, BEM is still applicable with a little modification. In this case, however, the method must be expanded to deal with the equivalent volume currents. The field produced by the volume currents is small compared to that of the surface currents, and for many practical problems it can simply be neglected. If, however, volume currents are significant, the regions changing rapidly from an unsaturated to a saturated state need to be separated. In 2D/RS and 3D this is accomplished by generating subareas and subvolumes in these respective regions. The equivalent volume currents are found by an iterative scheme and are put in the right hand side of the system of equations, rather than in the system matrix. This is a very important distinction as other methods, that deal with non-linear problems using integral equations, require that the volume unknowns appear in the system matrix which results in a volume integral formulation. Volume integral formulations usually result in significantly larger matrices and longer solution times.

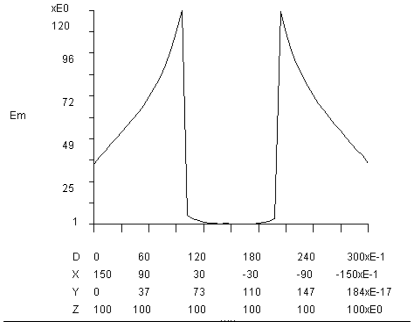
Accuracy of Three Dimensional Analysis

The design of a high voltage spark gaps has been carried out using both the two and three dimensional BEM electrostatic analysis packages, thus enabling a direct comparison to be made[3]. The example used was a water insulated self breakdown spark gap switch which employed a tailored PVC shank to confine the equipotentials, thus ensuring a uniform E-field in the de-ionised water region with reduced capacitive loading. The 3 dimensional geometry is depicted in Fig.6 and the electric field profile across the top of the PVC shank and through the lower electrode is shown in Fig.7. Further analysis including a comparison of the equipotential distribution using the two methods can be made through Ref.3. The E-fields in Figs.7 & 8 are in V/m for a normalised potential of 1V. This would equate to a peak E-field across the top of the insulator of 470kV/cm for a real potential of 388kV and of 466kV/cm for the lower switch surface flashover potential (~600kV/cm for switch design potential of 500kV). For slow rising or dc waveforms such an E-field may result in surface flashover and therefore a faster pulse must be applied to the high voltage electrode to ensure that the main breakdown would always occur between the two main electrodes. Such flashover limits should always be considered when designing spark gaps as this can be an area which gives concern, especially with respect to insulator lifetime in repetitive applications. The E-field in the inter-electrode gap was found to peak dramatically at 1000kV/cm at the earth tip. This field would ensure that breakdown would be initiated in the main spark region and that the statistical delay and therefore the delay to breakdown would be minimised for applied voltage pulses of a few 100 ns.



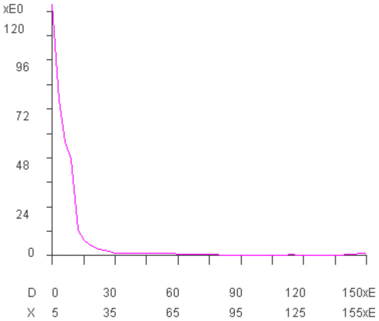
3-D water switch geometry

Fig.6



E-field across top of PVC shank

Fig.7



E-field on switch lower surface

Fig.8

The radial electrical field distribution along the bottom surface of the switch assembly from the point where the earth rod enters the water container and outwards was calculated[4] using both the two dimensional BEM Electro and the three dimensional BEM Coulomb (Fig.8). A direct comparison of the two analyses shows that they are in agreement to within ±3%. This comparison was carried out in an area of the switch with few elements in the 3D analysis since it was not an area critical to the switch performance. If the elemental structure was increased, the results of the analyses would converge thereby minimising the difference between the 2D & 3D solutions. A comparison of some of the different types of electromagnetic field simulation available for pulsed power engineers is given in Fig.9. This figure compares method of analysis and geometric representation of the problem, solution time and error detection processes.

Conclusions

The boundary element method offers a practical tool for pulsed power engineers to make electrostatic analysis of components and systems used in the generation of high voltage, high current pulses. As this method defines the problem from the boundaries (surfaces of conductors and insulators) then this allows a very accurate value for the fields in these areas to be determined. In practice these areas will tend to be most susceptible to unwanted discharges and therefore an accurate knowledge of the field values experienced on the surface during operation is of great practical benefit.

MODELING (3D) / ANALYSIS	IES BEM	CONVENTIONAL BEM	FEM
Physical Geometry	Exact geometry	Linear or quadratic fit	Artificial discontinuities (at element edges)
1. Mesh	Surface - easy production and inspection (N^2)	Surface - easy production and inspection (N^2)	Volume - complex to produce, inspection difficult (N^3)
2. Open Boundary Conditions	Automatically satisfied - for source balance or zero potential at infinity	Automatically satisfied - for zero potential at infinity	Artificial boundaries
3. Non-Linear Material	Subareas only where saturation occurs - general solution	Layered surface mesh	Unique values for each element
4. Solution Time	kN^4 diagonally strong matrix	N^6 full matrix	$N^{3.5 \text{ to } 4.5}$ sparse matrix
5. Error Detection	Simple	Simple	Complex

BEM versus FEM - Summary
Fig.9

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