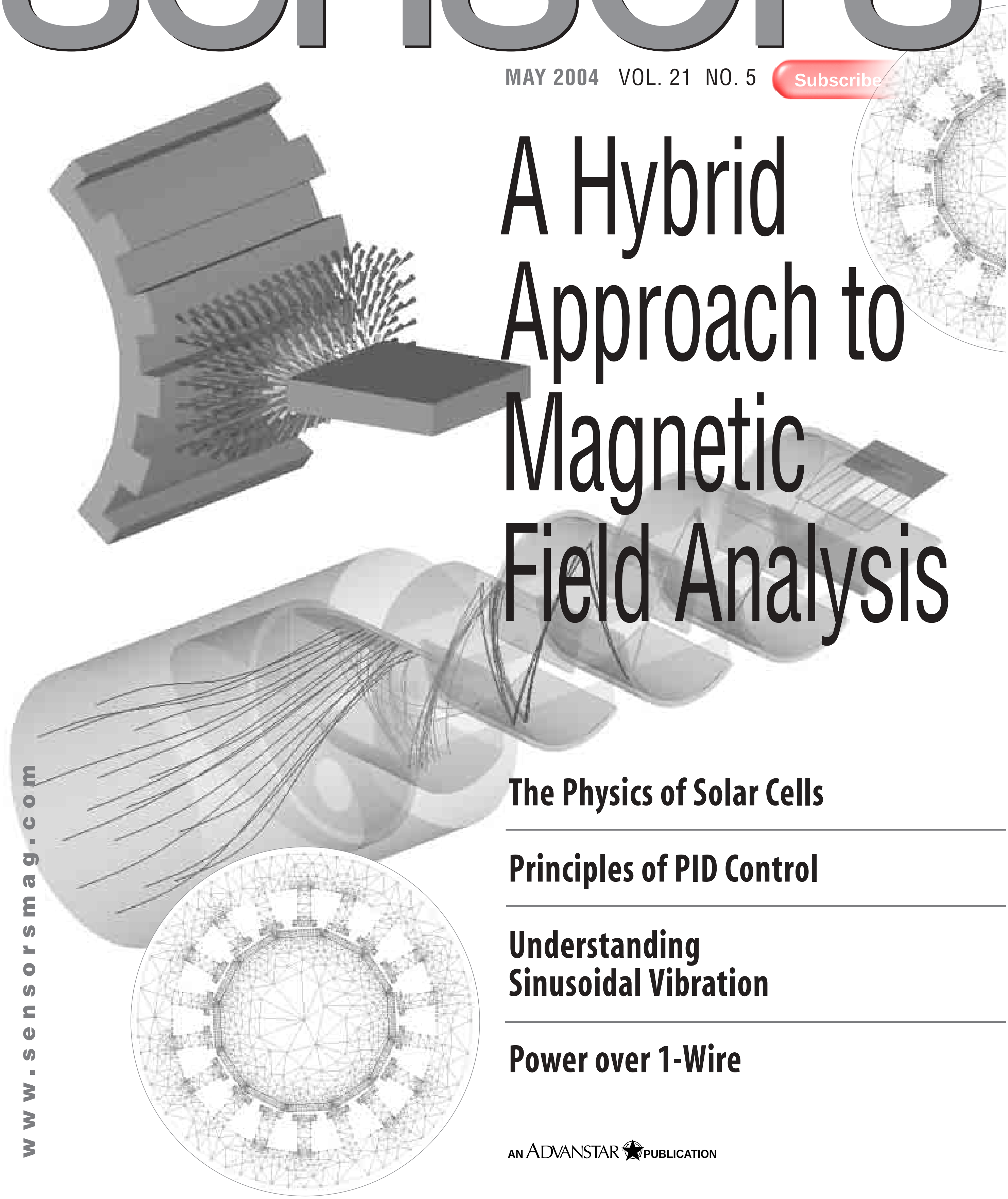
YOUR TECHNICAL RESOURCE FOR SENSING, COMMUNICATIONS, AND CONTROL

sensors

MAY 2004 VOL. 21 NO. 5

Subscribe



A Hybrid Approach to Magnetic Field Analysis

The Physics of Solar Cells

Principles of PID Control

Understanding Sinusoidal Vibration

Power over 1-Wire

www.sensorsmag.com

AN ADVANSTAR PUBLICATION

A Hybrid Magnetic Field Solver: Using a Combined Finite Element/Boundary Element Field Approach

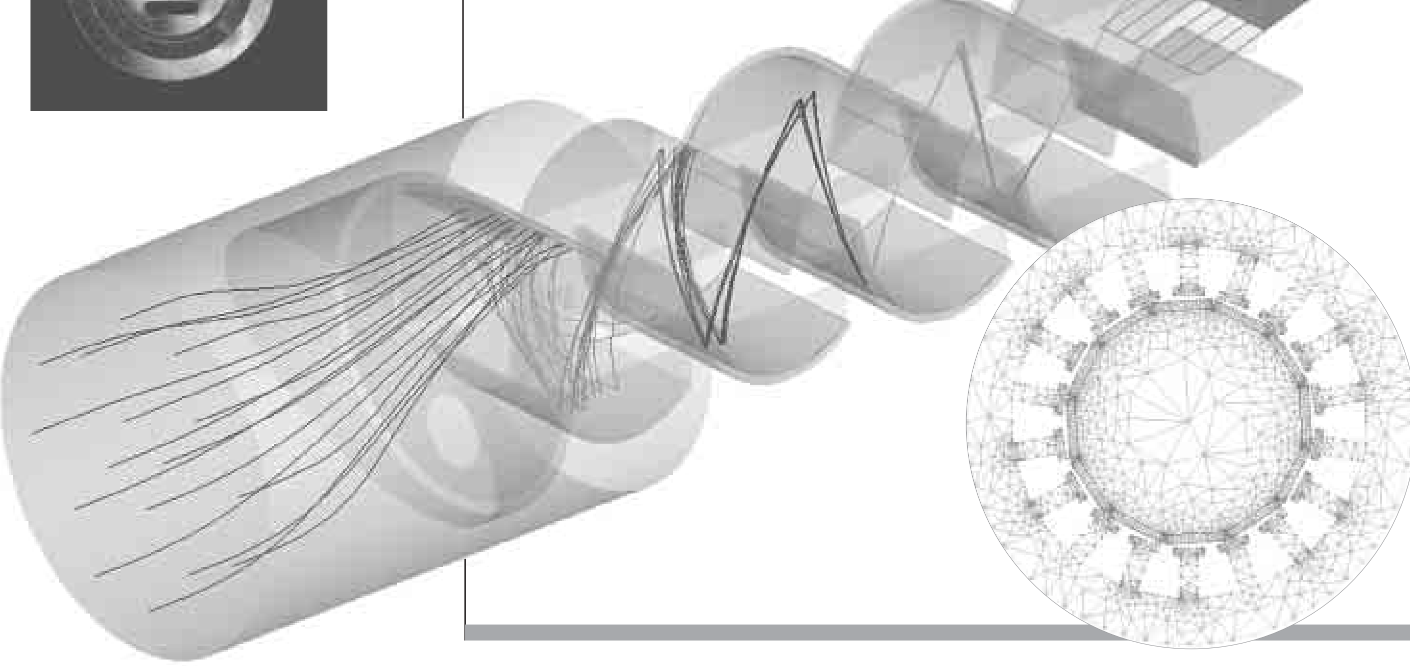
Magnetic analysis is a key component of motor design. Here's an explanation of a hybrid finite element/boundary element approach to solve for the magnetic fields involved.

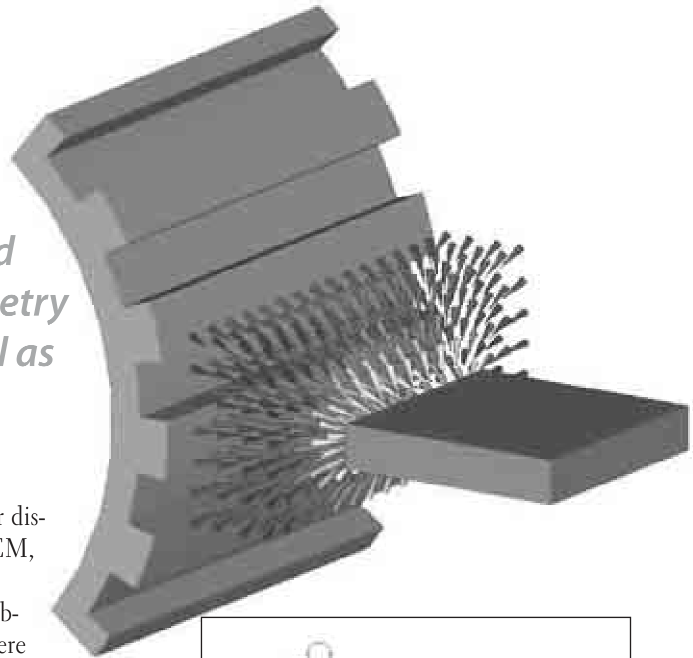
Bruce Klimpke, Integrated Engineering Software/Enginia Research

You can use two fundamentally different approaches to solve magnetic field problems. Both involve solving Maxwell's equations, but you can choose to solve them in either integral or differential form. The most common methods for solving problems in differential form are the finite difference (FD) method and the finite element method (FEM). Because of fundamental limitations of the FD approach, FEM is usually the preferred technique.

The second approach is to solve Maxwell's equations in integral form using either a boundary element method (BEM) or a volume integral method. We'll restrict the discussion to the BEM approach, as this is more typically used.

Both FEM and BEM have advantages and disadvantages, depending on the geometry and material properties involved, as well as the required accuracy. As a result, it's generally advantageous to combine differential and integral equation solvers to take advantage of their strengths to solve a given field problem. We'll use some real-world problems to show the advantages of each method and why a hybrid of the two is a better solution for some classes of problems.





Both FEM and BEM have advantages and disadva ntages, depending on the geometry and material properties involved, as well as the required accuracy.

Comparison of FEM/BEM Methods

For simplicity, we'll consider the following 2D problem as a basis for discussing some of the latent advantages and disadvantages of the FEM, BEM, and hybrid approaches (see Figure 1).

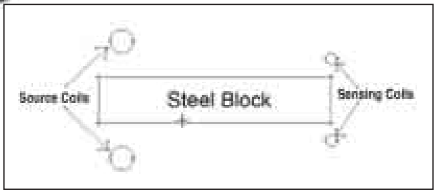


Figure 1. Here's our sample problem, which we'll be using to illustrate the various methods.

To solve a problem with finite elements, you have to discretize the problem's entire domain (see Figure 2). Note that the field doesn't end where the finite element mesh does; the real problem can never be completely modeled because the domain has to be truncated at some artificial location. The question is, how large must the enclosing boundary be in order to get a good approximation of the real problem? Also, what should the boundary condition be on that boundary? The simplest method is to set a zero potential boundary condition on the bounding surface. Other methods can sometimes be used to reduce the effect, truncating the region, but this is a limiting factor for electromagnetic problems.

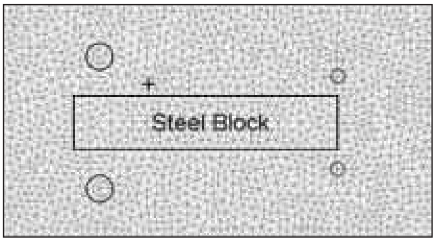


Figure 2. Here's the sample problem discretized using finite elements. Notice the artificial boundary to contain the finite elements.

Also note that the solution is the magnetic vector potential. Generally this parameter is of little interest, and what is really desired is the magnetic field density or intensity. To calculate the magnetic field density (**B**), you must differentiate the vector potential (**A**).

$$\mathbf{B} = \nabla \times \mathbf{A} \tag{1}$$

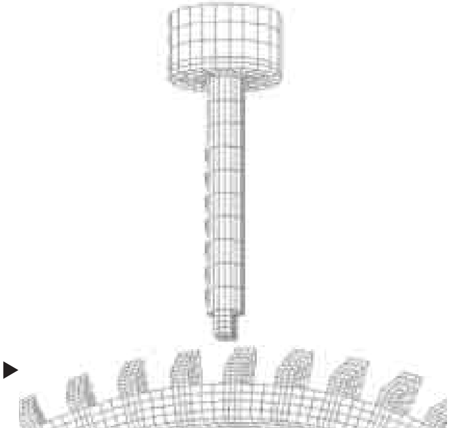
In Figure 2, each of the triangles is a finite element, and the points where they join are referred to as nodes. If the basis functions $\underline{\alpha}$ are nodal basis functions, then the A_i are the values of the magnetic vector potential at each of the nodes. For a vector potential in 3D, there will be three values at each node corresponding to each of the Cartesian coordinate directions. In 2D there is one unknown at each node point.

From a numerical perspective, differentiation is always an inherently unstable operation. Although the solution for **A** may appear to be quite good, its derivative will always be considerably worse.

The physical meaning of Equation (2):

$$\mathbf{A}(\mathbf{r}) = \int_s \mathbf{G}(\mathbf{r},\mathbf{r}')\mathbf{K}(\mathbf{r}')\,ds' + \int_v \mathbf{G}(\mathbf{r},\mathbf{r}')J(\mathbf{r}')\,dv' \tag{2}$$

can best be described with the aid of our original 2D drawing (see Figure 3).



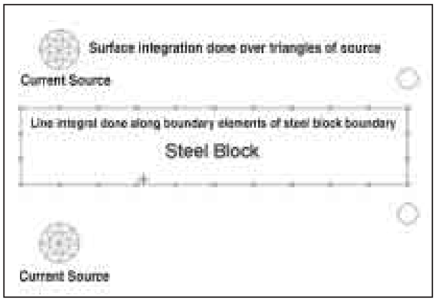


Figure 3. Here's the same sample problem, but discretized using boundary elements. Note that the exterior region has not been truncated or meshed.

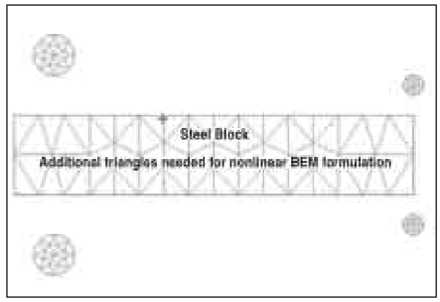


Figure 4. The mesh must be included in regions with nonlinear materials, as shown here.

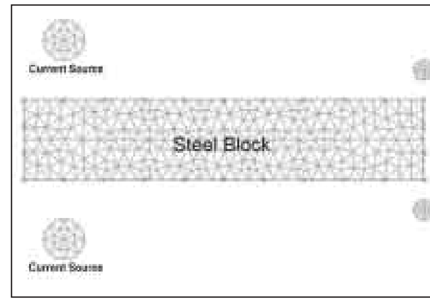


Figure 5. Here's the sample problem using FEM for the steel block and BEM for the remainder of the problem.

Part of the contribution to the value of $\mathbf{A}(\mathbf{r})$ in 2D is calculated by evaluating the surface integral in Equation (3), where $J_z(\mathbf{r}')$ is known and is part of the source term. The remaining contribution is calculated by evaluating the line integral in Equation (4).

$$\int_s G(\mathbf{r},\mathbf{r}')\mathbf{J}_z(\mathbf{r}')ds' \tag{3}$$

$$\oint_c G(\mathbf{r},\mathbf{r}')\mathbf{K}_z(\mathbf{r}')dl' \tag{4}$$

The analogy in 3D would be a volume and surface integral. The actual formulation to solve for $\mathbf{K}(\mathbf{r}')$ is not included here, but a Galerkin approach (similar to the FEM) can be used to define an inner product. In the BEM, the inner product enforces the continuity of $\mathbf{H}$ across a boundary:

$$\mathbf{n}\times(\mathbf{H}_2-\mathbf{H}_1)=\mathbf{K} \tag{5}$$

The details of the boundary element formulation have been omitted and can be found in many sources. A couple of key points should be noted about boundary elements.

First, you need discretize only the boundaries of the magnetic materials rather than the entire domain. This is true when the materials can be assumed linear. For nonlinear materials you may require additional unknowns within the volume.

Second, the region need not be artificially truncated. Theoretically the field can be calculated as far away from the problem as desired (including infinity).

Third, once $\mathbf{K}(\mathbf{r}')$ is known, the magnetic potential and magnetic density are calculated by integrating the unknown. The numerical process of integration is stable, resulting in complete continuity of the fields. The magnetic field density is calculated by integrating the unknown with the curl of the Green's function:

$$\mathbf{B}(\mathbf{r})=\int_s \nabla\times G(\mathbf{r},\mathbf{r}')\mathbf{K}(\mathbf{r}')ds' + \int_v \nabla\times G(\mathbf{r},\mathbf{r}')\mathbf{J}(\mathbf{r}')dv' \tag{6}$$

Let's discuss the major drawback of BEM. When using a finite element–based approach, dealing with nonlinear materials is straightforward. However, when using a boundary element–based approach and dealing with a material pushed highly into saturation, you lose the main advantage of having only to discretize the boundaries. In this case additional unknowns are required in the volume of the nonlinear material. So for our sample problem the discretization would now appear as in Figure 4. These additional volume calculations are expensive from a calculation point of view.

Hybrid Method

The hybrid approach combines BEM and FEM, taking advantage of the strengths of each method to solve a specific problem.

The approach is straightforward, but the implementation is quite difficult. First we have to decide which method to use in each volume (region in 2D): either FEM or BEM. The two methods are then tied together by enforcing the continuity conditions on either $\mathbf{B}$ or $\mathbf{H}$ at the boundaries. In general the strategy is to use boundary elements in all linear regions and finite elements in all nonlinear regions. In some instances, however, it's desirable to use finite elements in linear regions as well. The rule of thumb is if the surface area of a volume is large relative to the volume it encloses, and moderate solution accuracy is acceptable, finite elements may be the best choice for these regions. For our original model the steel would contain finite elements and the remainder of the problem would be solved using boundary elements (see Figure 5). Remember that the subdivisions or 2D elements in the coils are not finite elements but are simply there to do the surface integration:

$$\int_s G(\mathbf{r},\mathbf{r}')\mathbf{J}_z(\mathbf{r}')ds' \tag{7}$$

If for some reason we chose to use finite elements for the whole problem (except the region outside some bounding box, where we would use boundary elements), then the discretization would appear, as in Figure 6.

In both of these models the exterior region is not artificially truncated. If you require very high accuracy in all the exterior space, choose the first hybrid model. If speed is of primary concern, choose the second hybrid model.

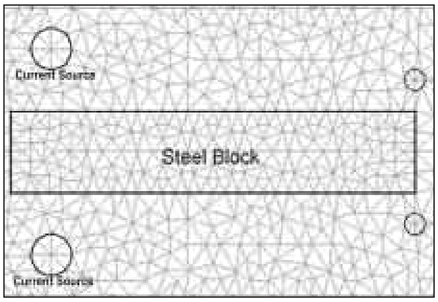


Figure 6. Here's the entire problem solved using FEM, except for the exterior region, where BEM is used.

Conclusion

The purpose of this article is to illustrate that the best method to use depends on the problem to be solved. In general the best strategy is to use a hybrid solver, in which the advantages of each method can be applied to each region for the entire problem.

As with all numerical techniques, however, there are always subtle difficulties with each method. Although basic guidelines can be established on which method to use, experience determines the one that will give the desired accuracy in the least amount of time.

For Futher Reading

Volakis, J.L., A. Chatterjee, L.C. Kempel, "Finite Element Method for Electromagnetics," Institute of Electrical & Electronics Engineers Inc., New York, NY, 1998.

Yildir, Y.B., B.W. Klimpke, D. Zheng, "A Computer Program for 2D/RS Eddy Current Problems Based on the Boundary Element Method," International Coil Winding Conference, Cincinnati, OH, September 1992.

Yildir, Y.B., B.W. Klimpke, K.M. Prasad, "Magnetic Analysis of Brushless DC Motors Using the Boundary Element Method," PCIM'91 Conference, Universal City, CA, September 1991. ■

Bruce Klimpke is Technical Director, Integrated Engineering Software/Enginia Research, Winnipeg, MB, Canada; 204-632-5636, bklimpke@integratedsoft.com, www.integratedsoft.com.

Four Sample Problems

Of the following four sample problems, one is ideally suited for BEM, one for FEM, and two for the hybrid method.

The first problem (see Figure 7) is a magnet testing system consisting of a magnet and measure coils. It's ideally suited for boundary elements. Magnet field density values of <10 ppm can be attained with very few elements (~20). Thousands of finite elements would be needed to solve this problem with relatively coarse field values.

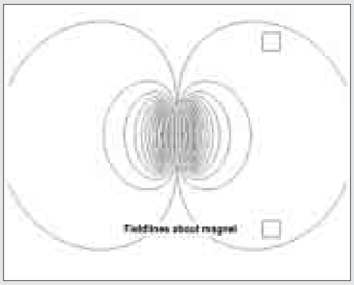


Figure 7. This is an ideal problem for a pure BEM approach, as the materials involved are linear, there is extensive free space, and high accuracy is required.

The second problem (see Figure 8) is a highly saturated induction motor. This problem is suited for finite elements because the solution in the exterior space is irrelevant and many parts of the problem are highly saturated. The volume area of the linear region is very small relative to the rest of the problem. This would be the worst case scenario for boundary elements. Also, because the torque computation involves the interaction of two sources, you can achieve accurate torque computations without very precise field computations. This may not hold for certain cogging torque applications.

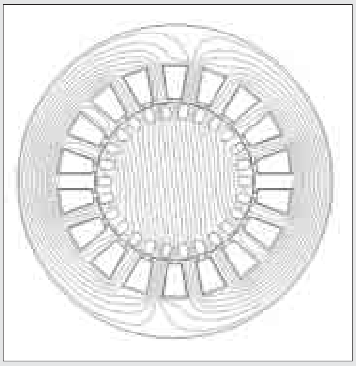


Figure 8. FEM would be suited for this problem because the exterior region is not important, and almost the entire problem is nonlinear.

The third problem (see Figure 9) is a magnetic structure for focusing a magnetic field. The problem involves a highly saturated field in the iron and requires highly accurate field computations in the air regions. This problem is ideally suited for a hybrid solution where boundary elements are used to calculate the field in the air space and finite elements are used in the nonlinear iron.

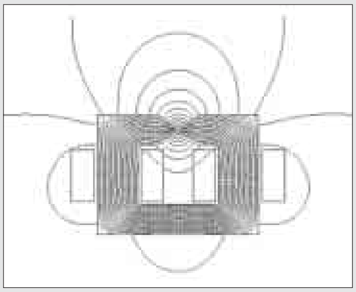


Figure 9. Here, highly accurate field computations are required in the space around the nonlinear structure. As a result, the hybrid method is the right approach.

The fourth problem (see Figure 10) is a cogging torque problem for a DC motor. These tend to be the most difficult problems for which to get meaningful results. Although the hybrid approach is expensive in terms of time, it's the most reliable method for computing cogging torques. It's also the best method if you want to accurately calculate the field exterior to the motor. □

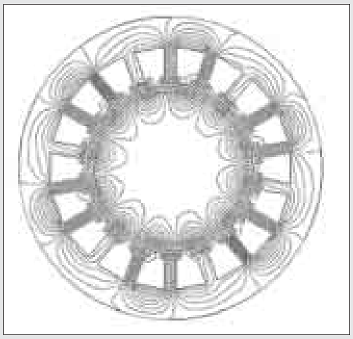


Figure 10. Cogging torque often requires accurate field computations. Although slow, the hybrid method will properly calculate the field in the space between the rotor and stator for the cogging torque calculation.

COMPARISON OF METHODS

Method	Advantages	Disadvantages
FEM	Easily applied to all types of problems, unlike BEM, where a different kernel or Green's function may be required for each domain of the problem.	Can be extremely inefficient if it can be applied at all for problems where a huge amount of volume must be discretized relative to the total surface area.
	Readily handles nonlinear problems of all types. Nonlinear regions must be discretized for both differential and integral formulations, and you can use very efficient nonlinear methods within the finite element system of equations.	Differentiating the solution to get the field is a numerically poor process; e.g., to calculate **B**, we have to take the curl **A**. Plots of **A** will appear smooth, but **B** will have discontinuities, artifacts of the differentiation.
	As long as a good mesh can be created, implementation is straightforward.	A large amount of data must be stored compared to that needed for BEM.
	Once the problem has been solved, it's trivial to calculate the field. Since the solution is known at the mesh nodes, you can interpolate to any point within the mesh.	The "action at a distance" concept applies only to equivalent source methods. For example, forces and torques can't be calculated by Amperian currents using FEM.
	Solution time can be better than with BEM for problems where the surface is large compared to the volume it contains.	Inherently integral equations are difficult to implement for general curved surfaces because of the singular kernel.
BEM	You can attain very high accuracy for fields because the field is calculated by integrating the solution.	Not easily applied to all types of problems. A different domain or Green's function may be required for each domain of the problem.
	The problem doesn't have to be artificially truncated, nor a boundary condition applied to the artificial boundary.	Cannot handle nonlinear problems efficiently. For weakly nonlinear problems the method is satisfactory, but for highly nonlinear problems the solution time is excessive.
	For linear problems, unknowns are located only on the boundaries of the problem. This radically reduces mesh generation time and storage requirements.	Longer solution time for post-processing calculations for some classes of problems when compared to FEM.

©Reprinted from SENSORS, May 2004 AN ADVANSTAR PUBLICATION Printed in U.S.A.