Benchmark Problems for Simulating Electric Fields Near Triple Junctions

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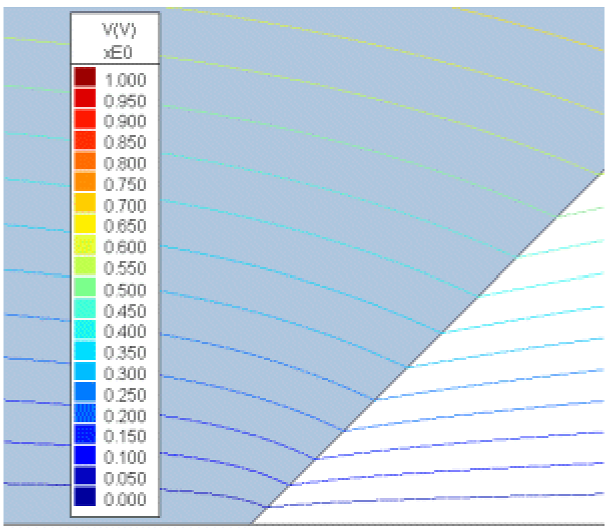
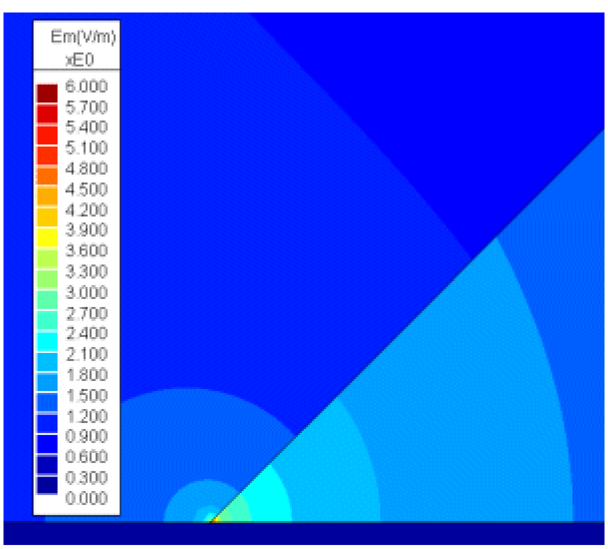
The Problem



One of the most common questions in Electromagnetic simulation is "what/where is the maximum field?". When a junction is present the question becomes tricky. In the sample case to the left the "triple junction" is a dielectric (left), air (right) and an electrode (bottom). Although the voltage looks well behaved, this junction is mathematically a singularity.

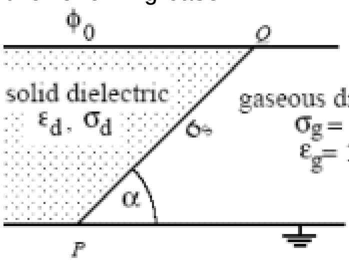


You cannot ask what is the direction of the field at the junction - that depends on the direction from which you approach it. Furthermore, the magnitude of the field is infinite at the corner. Hence, examining the field near a corner should produce increasingly high fields. The question posed is not meaningful because it depends on how close and from which direction you approach the corner. The danger is that your intuition which says real fields are not infinite may lead you to accept a bad answer.

In order to be confident you are getting correct answers you need to begin with a benchmark that can determine whether a given solver is producing the correct answer for the given question.

Solution by Boonchai Techaumnat et al

In Effect of Conductivity on Electric field Behaviour Near a Contact Point, Boonchai et al present the following case:

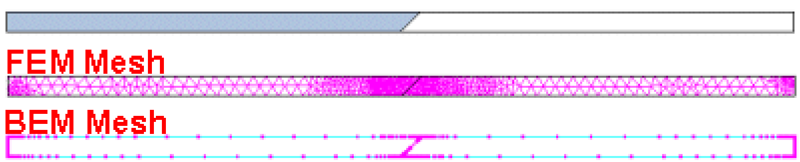


$$\begin{aligned}\phi_g &= a_1 r^{z_1} \sin z_1\theta \\ &\quad + a_2 r^{z_2} \sin z_2\theta + \cdots \\ \phi_d &= b_1 r^{z_1} \sin z_1(\pi - \theta) \\ &\quad + b_2 r^{z_2} \sin z_2(\pi - \theta) + \cdots\end{aligned}$$

Since the expansion of the potential near the point P is a superposition of terms of r^{number}, the closer you get to the point the more one term will dominate the solution. So, they use the boundary condition along the dielectric/air interface to determine the possible *number*s and choose the smallest as the one that will be dominant. They find E from the gradient of the potential equation and normalize by the constant value of the field far away. The results are remarkably close to their numeric simulations.

Sample Case

Based on Boonchai et al's findings we setup the following problem in our 2D electric field solver ELECTRO:



The model consists of a 1 m thick x 20 m long dielectric with relative permittivity=4 and conductivity=0. On top of the dielectric is a 40 m long 1 V electrode and underneath is a 40 m long 0 V electrode. To force the interior to become semi-infinite at the ends a linear voltage condition of 0-1 V is applied to the two 1 m tall ends.

The meshes shown are for two different analysis options in ELECTRO which will be benchmarked below.

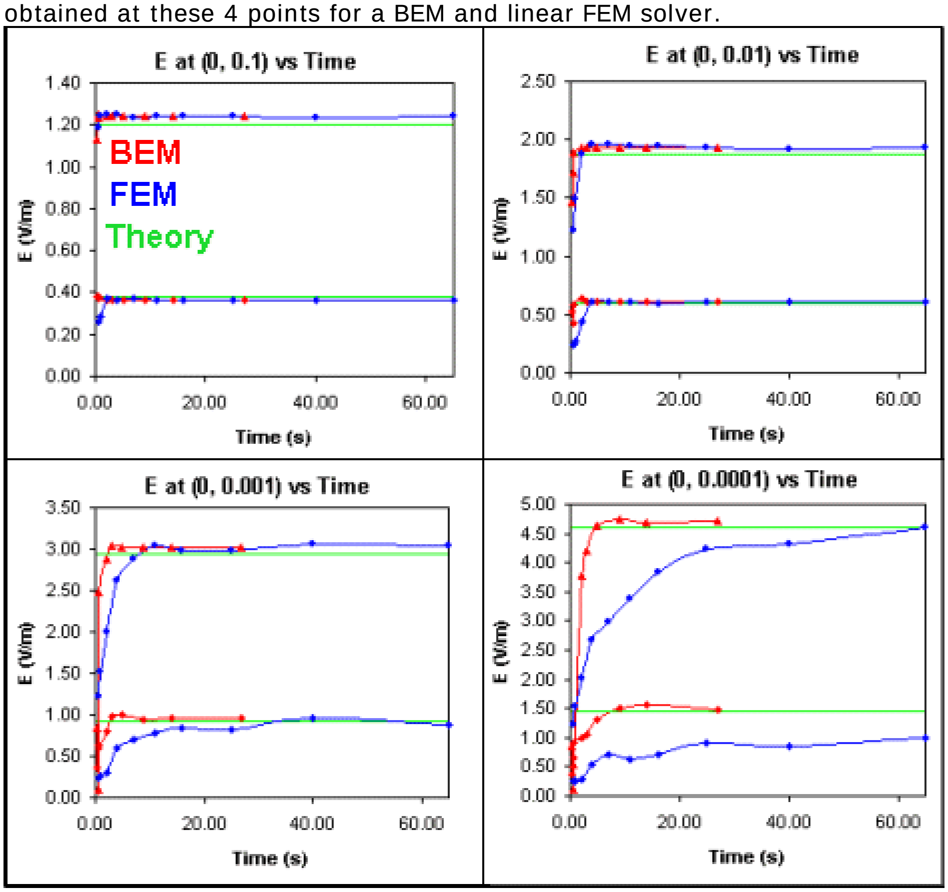
In this problem the possible values of *number* are real. To 6 decimal places the first few are 0.80603, 2.00000, 3.19397, and 4.80603. For small r the value 0.80603 will be dominant. Normalizing by 1 V/m far from the contact point, we get the E field in the dielectric near the point P as:

E_r = 0.80603 * $r^{(0.80603-1)}$sin(0.80603*theta)
E_{theta} = 0.80603 * $r^{(0.80603-1)}$cos(0.80603*theta)

To examine the speed and accuracy of ELECTRO's solvers four analysis points are chosen:

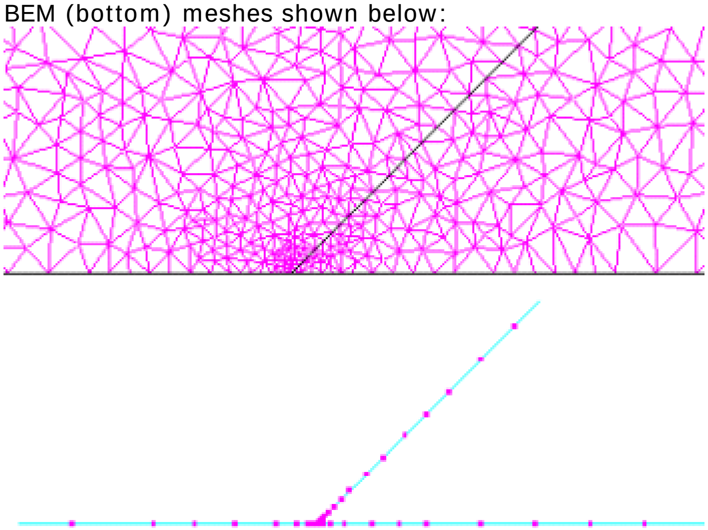
(x, y)	**E_r**	**E_{theta}**
(0, 0.1)	1.2018 V/m	0.3780 V/m
(0, 0.01)	1.8785 V/m	0.5908 V/m
(0, 0.001)	2.9362 V/m	0.9234 V/m
(0, 0.0001)	4.5894	1.4433

The first point should be easy to solve, but as the analysis point gets closer and closer to the triple junction singularity the analysis will become more difficult. The graphs below show a comparison of the effect of various solver accuracy settings on the solution time and value obtained at these 4 points for a BEM and linear FEM solver.



In both cases a high accuracy setting (longer analysis time) is required to obtain E values near the junction. Note that for high enough accuracy settings both solvers approach the results predicted by Boonchai et al's theory within a few percent, which is good considering the approximations of the theory and model. **However, the BEM solver approaches the value more rapidly and the more challenging the problem the more the BEM solver outperforms the FEM solver.** The reason is easy to understand. Consider the FEM (top) and

BEM (bottom) meshes shown below:



With both solvers the mesh is refined to create smaller elements where the field is varying the most rapidly - near the junction. The more accurate you request the solution, the smaller the elements near the junction will become. The spatial resolution of E in both cases is approximately the local size of the elements. With BEM, since the mesh is 1D if you create twice as many elements you get twice as good spatial resolution, if you put on 10 times as many elements you get 10 times the spatial resolution, if you put on 100 times as many elements you get 100 times the spatial resolutions, etc. However, with FEM, since the mesh is 2D if you create twice as many elements the spatial resolution improves by a factor of sqrt(2)=1.4, if you put on 10 times as many elements the spatial resolution improves by a factor of sqrt(10)=3, if you put on 100 times as many elements the spatial resolution improves by a factor of sqrt(100)10, etc.

So, the ability to improve spatial resolution by adjusting the element size is much more limited with FEM than with BEM and the difference shows up more significantly the more resolution you try to obtain. In fact, examining the blue (FEM) curves for the (0, 0.0001) case above, if you didn't know the correct answer you could easily be fooled into thinking the values had converged too early.

Summary

A simple to setup problem type for benchmarking solutions to electric fields near triple point junctions was proposed by Boonchai et al. Here we have run one specific case to study the discrepant results that people often find between their BEM & FEM solutions. The following conclusions are demonstrated:

1. Provided the BEM & FEM element lengths are comparable size in the vicinity of the point queried, the electric field answer will be the same.
2. It is much easier to get small elements with BEM, hence near singularities such as a triple junction it will tend to predict higher fields.
3. Since the element size only shrinks as the square root of the number of elements with FEM, the slow trend in refining the solution with more elements can give the wrong impression that the value has converged.
4. Due to 1), 2) & 3) above, **the impression that there is some "maximum E" near the triple junction is usually a weakness in the element distribution used and is not an indication of any physical aspect of the model they created**

5. Due to 4) above, a person using numerical methods to find the "maximum E" near a triple junction needs to be careful about how the problem is defined and analyzed. They need to realize that apparently small details such as a tangent versus slightly flattened intersection of two materials can give a much different conclusion very near the junction. They also need to know some reasonability criteria such as how close to the junction is significant or real to probe given the other approximations/assumptions of the model.

The computer used in this study was running Windows 2000 at 2.4 GHz.